5.2 Vector geometry proofs

Name: _____ Class: _____ Date: _____ **Total: 20 marks**

KEY WORDS resultant

Vector geometry proofs

Objective

Prove midpoint, parallel and ratio facts by writing one vector two independent ways, including unknown ratios solved by equating coefficients (objective Higher 5.1 G).

1 Key knowledge and worked examples

The method. Name a target vector, express it along TWO different routes, and equate. Parallel follows from $\overrightarrow{PQ} = c\,\overrightarrow{RS}$ (scalar multiple); a midpoint from $\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AB}$. Never assume a midpoint the question has not given: write "λ times" and solve for λ by equating the **a** and **b** coefficients.

Worked proof. $OABC$ parallelogram, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OC} = \mathbf{c}$, M the midpoint of diagonal OB. $\overrightarrow{OM} = \frac{1}{2}(\mathbf{a}+\mathbf{c})$ and $\overrightarrow{AM} = \frac{1}{2}(\mathbf{c}-\mathbf{a}) = \frac{1}{2}\overrightarrow{AC}$: the diagonals bisect each other.

2 Practice

2.1 In triangle OAB, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$; M is the midpoint of AB. Find $\overrightarrow{OM}$ [2]

2.2 State the condition on vectors **u** and **v** for their directions to be parallel. [1]

2.3 P divides $\overrightarrow{OQ} = 4\mathbf{a} + 6\mathbf{b}$ in the ratio $1:2$ from O. Find $\overrightarrow{OP}$ [2]

2.4 $\overrightarrow{XY} = 3\mathbf{a} - \mathbf{b}$ and $\overrightarrow{XZ} = 9\mathbf{a} - 3\mathbf{b}$. What does this prove about X, Y, Z? [2]

3 Exam-style questions

3.1 *(Original.)* $OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. (a) Write $\overrightarrow{OB}$ and $\overrightarrow{AC}$ in terms of **a** and **c**. (b) M is the midpoint of OB; prove that M is also the midpoint of AC. [5]

3.2 *(Edexcel 4MA1 2HR June 2024 Q25 family, adapted.)* The point P satisfies $\overrightarrow{OP} = \frac{8}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$ (the paper's part (i)). The point B satisfies $\overrightarrow{OB} = 2\mathbf{a} + \frac{3}{4}\mathbf{b}$, and M is the midpoint of OB. (a) Find $\overrightarrow{OM}$ in terms of **a** and **b**. (b) Show that $\overrightarrow{OM} = \frac{5}{8}\overrightarrow{OP}$ (the paper's k), and state what this proves about the points O, M and P. [4]

3.3 *(Original.)* In triangle ORS, T is the midpoint of OR and U is the midpoint of RS. Prove that $\overrightarrow{TU} = \frac{1}{2}\overrightarrow{OS}$ and state what this proves about the segment joining two midpoints. [4]