4.4 Circle geometry with reasons

Name: _____ Class: _____ Date: _____ **Total: 20 marks**

KEY WORDS cyclic quadrilateral • chord

Circle geometry with reasons

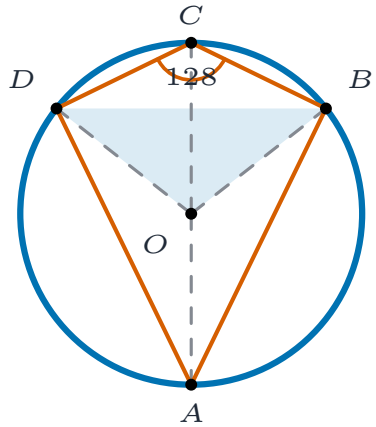
Objective

Use circle vocabulary and chord/tangent properties; at Higher, the cyclic-quadrilateral, centre/circumference and same-segment angle facts and the intersecting chord property – every step with a standard reason (objectives Foundation 4.6–4.7 and Higher 4.6–4.7).

1 Key knowledge and worked examples

The reason bank. Tangent $\perp$ radius at the point of contact; the perpendicular from the centre bisects a chord; angle at centre = 2 × angle at circumference (same arc); angles in the same segment are equal; the diameter subtends 90° at the circumference; opposite angles of a cyclic quadrilateral sum to 180°; triangle with two radii is isosceles. Quote the fact in words beside every numerical step.

Intersecting chords. Two chords meeting inside the circle satisfy $AE \times EB = CE \times ED$; the external version does the same from outside. If $AE = 6$, $EB = 8$, $CE = 4$, then $ED = 6 \times 8 \div 4 = 12$.



cyclic quad: $\angle BAD = 52$

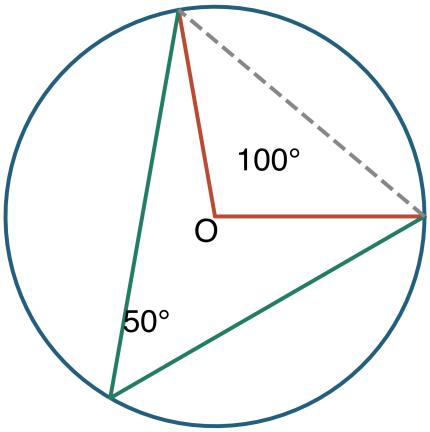
centre $= 2\times$ rim: $\angle BOD = 104$

$OB = OD$ (radii): isosceles

$\angle OBD = \frac{180-104}{2} = 38$

every numerical step names its reason

The centre angle is twice the circumference angle



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 Write down the angle at the centre when the angle at the circumference on the same arc is 41°. [1]

2.2 $ABCD$ is a cyclic quadrilateral with $\angle ABC = 97°$. Find $\angle ADC$. [2]

2.3 Two chords meet inside a circle: $AE = 6$, $EB = 8$ and $CE = 4$. Find ED. [2]

2.4 A tangent touches the circle at T and O is the centre. State the angle between the tangent and the radius OT. [1]

2.5 PQ is a diameter and R is on the circumference. Find $\angle PRQ$. [1]

3 Exam-style questions

3.1 *(Edexcel 4MA1 1HR June 2024 Q13, adapted.)* A, B, C, D are points on a circle centre O. $\angle BCD = 128°$. Find $\angle OBD$, giving reasons at each stage. [5]

3.2 *(Original.)* A, B, C lie on a circle centre O with $\angle AOB = 130°$. D lies on the major arc AB. (a) Find $\angle ADB$. (b) Find $\angle OAB$. [4]

3.3 *(Original.)* Chords PQ and RS intersect at X inside a circle with $PX = 5$, $QX = 9$, $RX = 6$. (a) Find SX. (b) The perpendicular from O to chord PQ meets it at M; state PM in terms of PQ and justify. [4]