3.6 Mixed Higher practice: sequences, functions and graphs

Name: _____ Class: _____ Date: _____ **Total: 32 marks**

KEY WORDS rate of change

Mixed Higher practice: sequences, functions and graphs

Objective

Combine the topic-3 strands in unfamiliar settings: series sums in context, perpendicular lines, composites and transformations together, intersections with exact answers, and calculus in motion (objectives Higher 3.1–3.4).

1 Key knowledge and worked examples

One line per strand. Arithmetic: $u_n = a + (n-1)d$ and $S_n = \frac{n}{2}[2a + (n-1)d]$. Functions: $\mathrm{fg}(x) = f(g(x))$, g first; invert a one-to-one function by rearranging. Lines: gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$; perpendicular gradient $-1/m$. Graphs: intersections solve $y_2 - y_1 = 0$; transformations $f(x) + a$, $f(x + a)$, $af(x)$, $f(ax)$ act as in the handout. Calculus: $\frac{d}{dx}ax^n = anx^{n-1}$; stationary where $\frac{dy}{dx} = 0$; motion has $v = \frac{ds}{dt}$, $a = \frac{dv}{dt}$.

2 Practice

2.1 Find a rule for the nth term of 6, 11, 16, 21, … [2]

2.2 Find the sum of the first 12 terms of $-3 + 1 + 5 + 9 + \cdots$ [3]

2.3 Find the equation of the line perpendicular to $y = \frac{1}{4}x - 7$ through the point $(5, 2)$. [3]

2.4 For $f(x) = \sqrt{3x - 5}$, write down the smallest value of x allowed in the domain. [2]

2.5 For $y = \frac{1}{4}x^4 - 2x^2$, find the x coordinates of all stationary points. [3]

3 Exam-style questions

3.1 *(Original.)* $f(x) = x^2 - 8x + 20$. (a) Write down the coordinates of the minimum point of $y = f(x)$. (b) The function g is defined by $g(x) = 2f(x-1)$. Find the coordinates of the minimum point of $y = g(x)$. [5]

3.2 *(Original.)* The line $y = 2x - 3$ meets the curve $y = x^2 - x - 6$. (a) Show that the x coordinates of the intersection points satisfy $x^2 - 3x - 3 = 0$. (b) Find the coordinates of both points, giving exact answers. [5]

3.3 *(Original.)* A theatre has 18 seats in row 1, and each row after it has 4 more seats than the row in front. (a) Find the number of seats in row 25. (b) Find the total number of seats in the first 25 rows. [5]

3.4 *(Original.)* A particle moves so that its displacement after t seconds is $s = 2t^3 - 15t^2 + 24t$ metres. (a) Find an expression for v in terms of t. (b) Find the values of t at which the particle is instantaneously at rest. [4]