6.6 Mixed practice: statistics and probability

Name: _____ Class: _____ Date: _____ **Total: 29 marks**

KEY WORDS independent events

Mixed practice: statistics and probability

Objective

Combine the topic-6 strands: density to frequency, grouped means with quartiles, ogive reads, Venn and complement shortcuts, and two-stage probability chains (objectives Foundation 6.1–6.3 and Higher 6.1–6.3).

1 Key knowledge and worked examples

One line per strand. Density is frequency over width – area is the frequency. Grouped mean by midpoints; quartiles by position; ogive reads at $n/2$, $n/4$, $3n/4$. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; complement for "at least one". Trees: multiply along, add between; without replacement adjusts the second branch.

2 Practice

2.1 A histogram bar of width 4 has density 3.5. Find its frequency. [2]

2.2 Find the mean of 6, 8, 10, 12, 14 [2]

2.3 Classes 0–6, 6–12, 12–24 hold frequencies 9, 10, 12. Which class has the highest density? [2]

2.4 P(rain) each independent day is 0.4. Find P(rain on exactly one of two days). [3]

2.5 Of 50 people, 30 speak French, 24 German, 10 both. Find P(a random person speaks neither). [3]

3 Exam-style questions

3.1 *(Original.)* A clinic records waiting times 0–10, 10–20, 20–30, 30–60 minutes with frequencies 8, 12, 15, 15. (a) Estimate the mean waiting time. (b) The manager claims "the typical wait is 30–60 minutes because that class holds the most". Comment. [5]

3.2 *(Original.)* For the clinic's data: (a) write the cumulative frequencies and plotting points; (b) estimate the median; (c) estimate how many patients waited more than 25 minutes. [6]

3.3 *(Original.)* A game: draw a counter from a bag of 3 gold, 5 silver, replace it, draw again. (a) Find P(at least one gold). (b) The rule changes to WITHOUT replacement: find P(at least one gold) now. (c) Explain why the answer increased. [6]

Solutions

2.1 $4 \times 3.5 = 14$.

2.2 $50/5 = 10$.

2.3 Densities: $9/6 = 1.5$, $10/6 \approx 1.67$, $12/12 = 1$ – the 6–12 class.

2.4 $0.4 \times 0.6 + 0.6 \times 0.4 = 0.48$.

2.5 Neither $= 50 - (30 + 24 - 10) = 6$; $P = 6/50 = 3/25$.

3.1 (a) Midpoints 5, 15, 25, 45: $(40 + 180 + 375 + 675)/50 = 1270/50 = 25.4$ minutes. (b) The claim confuses frequency with typicality – 30–60 TIES 20–30 on frequency (15 each), its density is far lower (0.5 vs 1.5), and the estimated mean is 25.4, inside 20–30. The wider class merely collects many patients across a long range.

3.2 (a) Cumulative frequencies 8, 20, 35, 50 at upper boundaries 10, 20, 30, 60 – the plotted points are $(10, 8), (20, 20), (30, 35), (60, 50)$. (b) The 25th patient lies between 20 (cumulative 20) and 30 (cumulative 35): $20 + 10 \times \frac{5}{15} \approx 23.3$ minutes. (c) At 25 minutes – halfway through the 20–30 class (width 10, frequency 15) – the cumulative frequency is $20 + 15 \times \frac{5}{10} = 27.5$, so about $50 - 27.5 = 22.5$: roughly 23 patients waited more than 25 minutes.

3.3 (a) With replacement: P(no gold) $= \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$, so P(at least one gold) $= \frac{39}{64}$. (b) Without: P(no gold) $= \frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \frac{5}{14}$, so P(at least one gold) $= 1 - \frac{5}{14} = \frac{9}{14}$. (c) After a non-gold counter is removed, 3 of the remaining 7 are gold – a better chance ($\frac{3}{7}$) than the $\frac{3}{8}$ with replacement – so the no-gold path got harder ($\frac{5}{14} < \frac{25}{64}$) and drawing at least one gold became likelier. Local review guidance: the correct strand tool chosen (density, midpoints, ogive read, complement); exact values with units; comment questions answered with a number, not an opinion. This is authored classroom guidance.