6.5 Tree diagrams and combined events

Name: _____ Class: _____ Date: _____ **Total: 26 marks**

KEY WORDS tree diagram • conditional probability

Tree diagrams and combined events

Objective

Draw and use tree diagrams, multiply along branches for independent events, handle without-replacement dependence, and read conditional probabilities from branches (objectives Higher 6.3 A–D).

1 Key knowledge and worked examples

Trees. Multiply along a path, add between paths. WITH replacement the second branch repeats (independent: $P(A \cap B) = P(A)P(B)$); WITHOUT replacement the second denominator drops by one and its numerator adjusts – show the changing fractions. Conditional: $P(B \mid A) = P(A \cap B)/P(A)$, read off the second branch.

The paper's shape. June 2024 1HR Q21 family: a bag with three colours, two draws without replacement, an unknown x – the two-route probability equals a given value, expanding gives a quadratic (the paper: $x = 11$, probability $\frac{7}{75}$). Reject the root the context forbids.

2 Practice

2.1 A coin is tossed twice. Draw or describe the tree and find P(two heads). [2]

2.2 A bag has 4 red and 6 blue counters; one is taken, NOT replaced, then a second. Find P(two red). [2]

2.3 For the same bag, find P(one of each colour). [3]

2.4 A spinner has P(win) = 0.2 each spin. Two independent spins: find P(exactly one win). [3]

3 Exam-style questions

3.1 *(Edexcel 4MA1 1HR June 2024 Q21 family, adapted.)* A bag holds 5 red, 6 blue and 9 green sweets. Two sweets are taken without replacement. (a) Find P(one red and one blue, in either order). (b) Find P(neither is green), and check it with the complement of "at least one green". [5]

3.2 *(Original.)* A box contains 3 batteries of which 1 is dead. Two are tested one after another WITHOUT replacement. (a) Draw the tree. (b) Find P(the dead battery is found by the second test). (c) Given the first tested battery works, find the probability the second is dead. [5]

3.3 *(Original, algebra.)* A bag contains 6 mints and x toffees. Two sweets are taken without replacement and P(both are mints) $= \frac{1}{3}$. (a) Show that $x^2 + 11x - 60 = 0$. (b) Find x, rejecting any impossible root. [6]

Solutions

2.1 Four equally likely paths; P(HH) $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

2.2 $\frac{4}{5} \times \frac{3}{9} = \frac{12}{45} = \frac{4}{15}$.

2.3 $\frac{4}{10} \times \frac{6}{9} + \frac{6}{10} \times \frac{4}{9} = \frac{48}{90} = \frac{8}{15}$.

2.4 $0.2 \times 0.8 + 0.8 \times 0.2 = 0.32$.

3.1 (a) $\frac{5}{20} \times \frac{6}{19} + \frac{6}{20} \times \frac{5}{19} = \frac{60}{380} = \frac{3}{19}$. (b) P(neither green) $= \frac{11}{20} \times \frac{10}{19} = \frac{110}{380} = \frac{11}{38}$; complement: P(at least one green) $= 1 - \frac{11}{38} = \frac{27}{38}$. ✓

3.2 (a) First branch: works $\frac{2}{3}$, dead $\frac{1}{3}$; second branch (2 left): after "works": works $\frac{1}{2}$, dead $\frac{1}{2}$; after "dead": works 1, dead 0. (b) $\frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times 0 = \frac{1}{3}$. (c) $P(\text{second dead} \mid \text{first works}) = \frac{1}{2}$ – the second branch after "works".

3.3 (a) $P(\text{both mints}) = \frac{6}{x+6} \times \frac{5}{x+5} = \frac{30}{(x+6)(x+5)} = \frac{1}{3}$. Cross-multiplying: $(x+6)(x+5) = 90$, so $x^2 + 11x + 30 = 90$, giving $x^2 + 11x - 60 = 0$. ✓ (b) $(x+15)(x-4) = 0$, so $x = 4$ or $x = -15$; a negative count is impossible, so $x = 4$. Check: with 4 toffees, $\frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$. ✓

Local review guidance: branch fractions adjusted for the removal; exact probabilities; algebraic parts end with a CONTEXT decision on the root. This is authored classroom guidance.