5.5 Mixed practice: vectors and transformations

Name: _____ Class: _____ Date: _____ **Total: 21 marks**

KEY WORDS translation

Mixed practice: vectors and transformations

Objective

Combine the topic-5 strands: vector routes with unknown ratios, moduli, transformation images and complete descriptions in unfamiliar settings (objectives Higher 5.1 A–G, Foundation 5.2 A–M).

1 Key knowledge and worked examples

One line per strand. Vectors: routes add tip-to-tail; reverse by negating; modulus by Pythagoras. Proofs: same vector two ways, then equate **a** and **b** coefficients. Transformations: name the type AND every parameter; the three isometries congrue the image, enlargements only make it similar.

2 Practice

2.1 Simplify $3\binom{2}{-1} - 2\binom{1}{4}$ [2]

2.2 Find the modulus of $\binom{-8}{6}$ [2]

2.3 Rotate $(2, -5)$ by $180°$ about O [1]

2.4 Enlarge $(-4, 6)$ by scale factor $\frac{1}{2}$ about the origin. [1]

2.5 $\overrightarrow{AB} = 5\mathbf{a} - 2\mathbf{b}$ and $\overrightarrow{BC} = -3\mathbf{a} + 2\mathbf{b}$. Find $\overrightarrow{AC}$ [2]

3 Exam-style questions

3.1 *(Original.)* $OAPB$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. (a) Write $\overrightarrow{AB}$ and $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (b) The point G satisfies $\overrightarrow{OG} = \frac{1}{3}\overrightarrow{OP}$. Find $\overrightarrow{AG}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (c) Explain how your answer shows that G does not lie on AB. [5]

3.2 *(Original.)* The point $Q(4, 3)$ is reflected in the y-axis to Q_1; Q_1 is then translated by $\binom{-1}{2}$ to Q_2; Q_2 is then rotated $90°$ anticlockwise about O to Q_3. Find Q_3. [3]

3.3 *(Original, transfer.)* A drone flies 6 km east then 8 km north (vectors from origin). (a) Write the two legs as column vectors. (b) Find the resultant displacement. (c) The drone then flies directly back to the start: write the return leg as a column vector and state its magnitude. [5]

Solutions

2.1 $\binom{6}{-3} - \binom{2}{8} = \binom{4}{-11}$.

2.2 $\sqrt{64+36} = 10$.

2.3 $(-2, 5)$.

2.4 $(-2, 3)$.

2.5 $\overrightarrow{AC} = (5\mathbf{a} - 2\mathbf{b}) + (-3\mathbf{a} + 2\mathbf{b}) = 2\mathbf{a}$.

3.1 (a) $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$; $\overrightarrow{OP} = \mathbf{a} + \mathbf{b}$. (b) $\overrightarrow{AG} = \overrightarrow{AO} + \overrightarrow{OG} = -\mathbf{a} + \frac{1}{3}(\mathbf{a} + \mathbf{b}) = \frac{1}{3}\mathbf{b} - \frac{2}{3}\mathbf{a}$. (c) If G lay on AB, then $\overrightarrow{AG}$ would be a multiple of $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$; matching coefficients would need the $\mathbf{a}$ and $\mathbf{b}$ coefficients of $\frac{1}{3}\mathbf{b} - \frac{2}{3}\mathbf{a}$ to be equal in size and opposite in sign, but they are $-\frac{2}{3}$ and $\frac{1}{3}$ – not a multiple of $\mathbf{b} - \mathbf{a}$. So G is not on AB (it lies inside the parallelogram, a third of the way along the diagonal).

3.2 Reflect in the y-axis: $(4, 3) \to (-4, 3)$. Translate by $\binom{-1}{2}$: $(-4, 3) \to (-5, 5)$. Rotate $90°$ anticlockwise about O: $(x, y) \to (-y, x)$, so $(-5, 5) \to (-5, -5)$. $Q_3 = (-5, -5)$.

3.3 (a) $\binom{6}{0}$ and $\binom{0}{8}$. (b) $\binom{6}{8}$ (km, east 6, north 8). (c) Return leg $= -\binom{6}{8} = \binom{-6}{-8}$; magnitude $\sqrt{36+64} = 10$ km.

Local review guidance: each transformation or route step applied in order; exact coordinates and vectors; conclusions state the geometric fact, not just the algebra. This is authored classroom guidance.