5.3 Transformations: rotation, reflection, translation

Name: _____ Class: _____ Date: _____ **Total: 15 marks**

KEY WORDS rotation • reflection

Transformations: rotation, reflection, translation

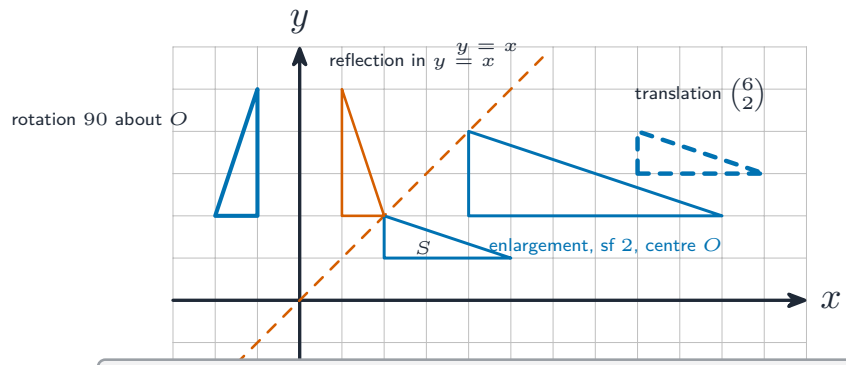
Objective

Rotate about a centre, reflect in a mirror line, translate by a column vector, and describe transformations completely; know that these three preserve length and angle so images are congruent (objectives Foundation 5.2 A–I).

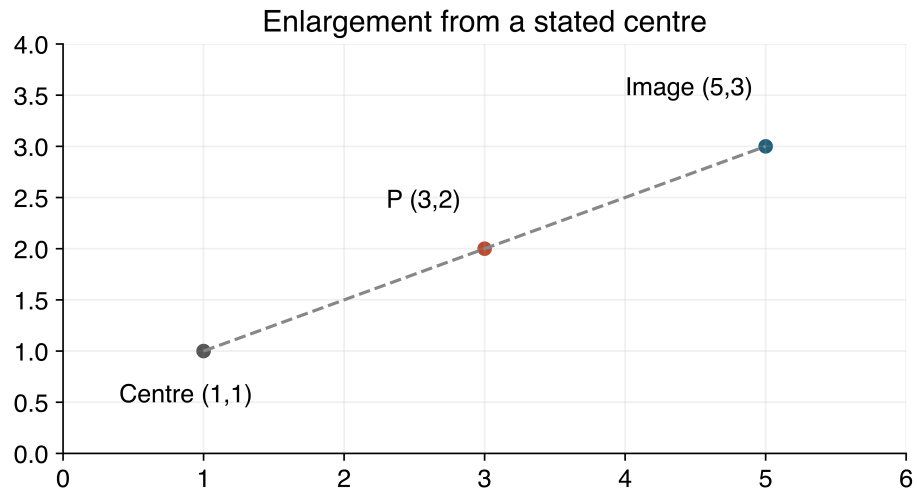
1 Key knowledge and worked examples

The three isometries. Rotation: centre + angle + direction (anticlockwise positive). Reflection: a mirror line such as $x = 1$, $y = 2$, $y = x$. Translation: distance and direction as the column vector $\binom{a}{b}$. All three preserve lengths and angles, so every image is congruent to the original.

Coordinate shortcuts. Reflect: x-axis $(x, y) \to (x, -y)$; y-axis $(x, y) \to (-x, y)$; $y = x$: $(x, y) \to (y, x)$. Rotate about O: $90°$ anticlockwise $(x, y) \to (-y, x)$; $180°$: $(x, y) \to (-x, -y)$. Translate: $(x, y) \to (x + a, y + b)$.



a complete description names the transformation AND every parameter



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 Reflect $(3, 5)$ in the x-axis. [1]

2.2 Reflect $(3, 5)$ in the line $y = x$. [1]

2.3 Rotate $(-2, 4)$ through $90°$ anticlockwise about O. [2]

2.4 Translate $(1, -3)$ by the vector $\binom{4}{6}$ [1]

2.5 A shape and its image have equal side lengths and equal angles, and every point has moved the same distance in the same direction. Name the transformation. [1]

3 Exam-style questions

3.1 *(Edexcel 4MA1 2HR June 2024 Q23 family, adapted.)* The point $P(2, 7)$ lies on the graph of $y = f(x)$. Write down the coordinates of the image of P on the graph of (a) $y = f(x - 4)$ (b) $y = 2f(x)$. [2]

3.2 *(Original.)* Triangle A has vertices $(1, 1)$, $(4, 1)$, $(1, 3)$. It is rotated $90°$ clockwise about $(0, 0)$. (a) Draw or describe the image's vertices. (b) Describe fully the single transformation that maps the image back onto A. [4]

3.3 *(Original.)* Shape B is the image of shape A under a reflection in the line $x = 2$. State the coordinates of the image of the point $(5, -1)$, and explain why the image is congruent to A. [3]

Solutions

2.1 $(3, -5)$.

2.2 $(5, 3)$.

2.3 $(-4, -2)$.

2.4 $(5, 3)$.

2.5 A translation.

3.1 (a) $y = f(x-4)$ shifts 4 right: $(6, 7)$. (b) $y = 2f(x)$ stretches vertically, factor 2: $(2, 14)$.

3.2 (a) $90°$ clockwise about O: $(x, y) \to (y, -x)$, giving $(1, -1)$, $(1, -4)$, $(3, -1)$. (b) Rotation, $90°$ anticlockwise about $(0, 0)$ (the inverse).

3.3 Reflection in $x = 2$: x-coordinates satisfy $\frac{x+x'}{2} = 2$, so $x' = 4 - 5 = -1$: the image is $(-1, -1)$. Reflections preserve lengths and angles, so the image is congruent to A.

Local review guidance: the correct rule or construction applied; exact coordinates; descriptions name centre/line/vector AND direction/factor. This is authored classroom guidance.