

5.1 Vector notation, resultants and moduli

Name: _____ Class: _____ Date: _____

Total: 23 marks

KEY WORDS vector • modulus

Vector notation, resultants and moduli

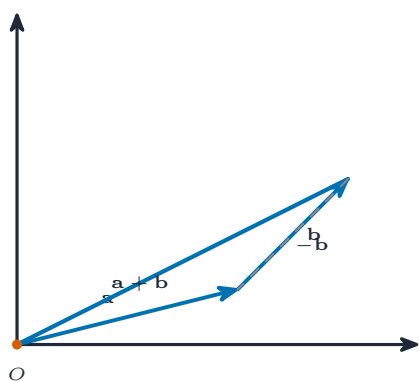
Objective

Use vector notation and column vectors, multiply by scalars, add and subtract componentwise, find resultants along routes and compute moduli (objectives Higher 5.1 A–F).

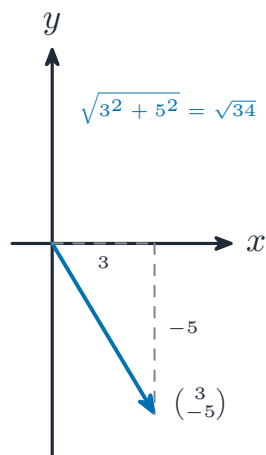
1 Key knowledge and worked examples

Vectors and routes. A vector has magnitude and direction: bold $\mathbf{a}$ or $\overrightarrow{OA}$ or the column $\begin{pmatrix} x \\ y \end{pmatrix}$. Scalars stretch: $2\mathbf{a}$ parallel and twice as long, $-\mathbf{a}$ reversed. Routes add tip-to-tail: $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$, and $\overrightarrow{BA} = -\overrightarrow{AB}$.

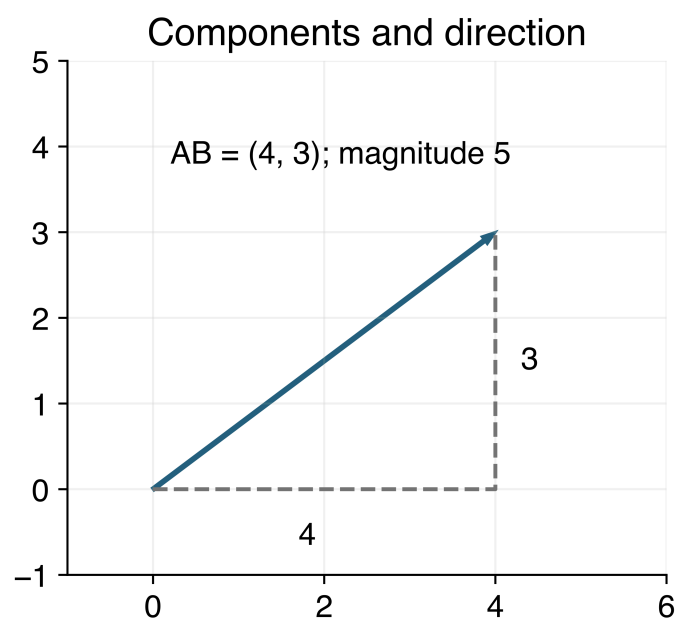
Modulus. Componentwise arithmetic, then Pythagoras for length: $\begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \end{pmatrix}$ and $\left| \begin{pmatrix} 3 \\ -5 \end{pmatrix} \right| = \sqrt{9 + 25} = \sqrt{34} \approx 5.83$.



tip-to-tail: the route adds, the stops do not



modulus = length, by Pythagoras



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 Write $\mathbf{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$. Find $\mathbf{a} + \mathbf{b}$

[2]

2.2 Find $2\mathbf{a} - \mathbf{b}$ for the same vectors. [2]

2.3 Find $|\mathbf{a}|$ for $\mathbf{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$. [2]

2.4 $\overrightarrow{OA} = 3\mathbf{a}$, $\overrightarrow{AB} = 2\mathbf{b}$, $\overrightarrow{BC} = 2\mathbf{c}$. Write down $\overrightarrow{OC}$ [2]

2.5 Write down $\overrightarrow{CO}$ in terms of $\overrightarrow{OC} = 3\mathbf{a} + 2\mathbf{b} + 2\mathbf{c}$. [1]

3 Exam-style questions

3.1 (*Original.*) P , Q , R are such that $\overrightarrow{PQ} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and $\overrightarrow{QR} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$. (a) Find $\overrightarrow{PR}$. (b) Find $|\overrightarrow{PR}|$ to 3 significant figures. [4]

3.2 (*Original.*) In triangle ABC , M is the midpoint of AB . $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$. (a) Find $\overrightarrow{AM}$. (b) Find $\overrightarrow{AC}$. (c) Show that $\overrightarrow{MC} = \overrightarrow{AC} - \overrightarrow{AM}$ and hence find $\overrightarrow{MC}$. [5]

3.3 (*Original.*) $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$. The vector $\mathbf{u} = 2\mathbf{a} + k\mathbf{b}$ has modulus $5\sqrt{2}$. Find both possible values of k . [5]

Solutions

2.1 $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$.

2.2 $2\begin{pmatrix} 4 \\ -3 \end{pmatrix} - \begin{pmatrix} -1 \\ 6 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \end{pmatrix} - \begin{pmatrix} -1 \\ 6 \end{pmatrix} = \begin{pmatrix} 9 \\ -12 \end{pmatrix}$.

2.3 $\sqrt{16+9} = 5$.

2.4 $\overrightarrow{OC} = 3\mathbf{a} + 2\mathbf{b} + 2\mathbf{c}$ (route rule).

2.5 $\overrightarrow{CO} = -\overrightarrow{OC} = -3\mathbf{a} - 2\mathbf{b} - 2\mathbf{c}$.

3.1 (a) $\overrightarrow{PR} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ 7 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$. (b) $\sqrt{16+25} = \sqrt{41} = 6.40$ (3 s.f.).

3.2 (a) $\overrightarrow{AM} = \frac{1}{2}\begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$. (b) $\overrightarrow{AC} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} + \begin{pmatrix} -2 \\ 8 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$. (c) The route $A \rightarrow M \rightarrow C$ gives $\overrightarrow{MC} = \overrightarrow{AC} - \overrightarrow{AM} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$.

3.3 $\mathbf{u} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 3k \\ k \end{pmatrix} = \begin{pmatrix} 2+3k \\ 4+k \end{pmatrix}$. The modulus condition gives $(2+3k)^2 + (4+k)^2 = 50$, so $4 + 12k + 9k^2 + 16 + 8k + k^2 = 50$, which simplifies to $10k^2 + 20k - 30 = 0$, then $k^2 + 2k - 3 = 0$. Factorising, $(k+3)(k-1) = 0$, so $k = 1$ or $k = -3$. Check $k = 1$: $\begin{pmatrix} 5 \\ 5 \end{pmatrix}$ has modulus $\sqrt{50} = 5\sqrt{2}$. ✓

Local review guidance: the route written before components; exact column answers; modulus statements show the Pythagoras step. This is authored classroom guidance.