4.7 Mensuration: perimeter, area, surface and volume

Name: _____ Class: _____ Date: _____ **Total: 28 marks**

KEY WORDS sector • volume

Mensuration: perimeter, area, surface and volume

Objective

Find perimeters and areas of composite shapes, circles and sectors; surface areas and volumes of prisms, cylinders, cones and spheres; convert metric units (objectives Foundation 4.9–4.10 and Higher 4.9, 4.10).

1 Key knowledge and worked examples

Area toolkit. Triangle $\frac{1}{2}bh$, parallelogram bh, trapezium $\frac{1}{2}(a+b)h$, circle πr^2 with circumference $2\pi r$. Higher sectors: arc $= \frac{\theta}{360} \times 2\pi r$, area $= \frac{\theta}{360} \times \pi r^2$ – for $r = 9$, $\theta = 40°$: arc $2\pi \approx 6.28$, area $9\pi \approx 28.3$. Composite shapes split into parts before any formula is applied.

Solids. Prism = cross-section × length; cylinder $V = \pi r^2 h$, surface $2\pi r^2 + 2\pi rh$; cone $V = \frac{1}{3}\pi r^2 h$, curved surface πrl with l the slant; sphere $V = \frac{4}{3}\pi r^3$, surface $4\pi r^2$. Unit chains: area 100^2, volume 100^3 between cm and m.

2 Practice

2.1 Find the area of a trapezium with parallel sides 7 cm and 11 cm and height 4 cm. [2]

2.2 Find the circumference and area of a circle of radius 5 cm, exact in terms of π. [2]

2.3 A sector has $r = 9$ cm and $\theta = 40°$. Find its arc length. [2]

2.4 Find the volume of a cylinder with $r = 5$ cm and $h = 10$ cm, in terms of π. [2]

2.5 A cone has $r = 3$ cm and $h = 4$ cm. Find its slant, volume and curved surface area, in terms of π where exact. [4]

3 Exam-style questions

3.1 *(Edexcel 4MA1 2HR June 2024 Q10, adapted.)* Hexagon $ABCDEF$ has $AB \parallel FC \parallel ED$ with $AB = 11$ cm, $ED = FC = 23$ cm, the perpendicular distance between FC and ED equal to 4.7 cm, and $BC = 5$ cm making a 30° angle with FC. Find the area of the hexagon. [5]

3.2 *(Original.)* A sphere has radius 6 cm. (a) Find its volume in terms of π. (b) Find its surface area in terms of π. (c) A cylinder has the same radius and the same volume; find its height. [5]

3.3 *(Original.)* A barn silo is a cylinder ($r = 2$ m, $h = 5$ m) topped by a hemisphere. (a) Find the total volume in terms of π. (b) Find the total surface area in terms of π. [6]

Solutions

2.1 $\frac{1}{2}(7+11)\times 4 = 36$ cm^2.

2.2 10π cm; 25π cm^2.

2.3 $\frac{40}{360}\times 2\pi\times 9 = 2\pi \approx 6.28$ cm.

2.4 $\pi\times 25\times 10 = 250\pi$ cm^3.

2.5 Slant $l = \sqrt{9+16} = 5$ cm; $V = \frac{1}{3}\pi\times 9\times 4 = 12\pi$ cm^3; curved surface $\pi\times 3\times 5 = 15\pi$ cm^2.

3.1 The strip between the parallels FC and ED is a rectangle 23×4.7; above it, the height of the trapezium $ABCF$ is $5\sin 30^\circ = 2.5$ cm. Split: rectangle $23\times 4.7 = 108.1$ plus trapezium $\frac{1}{2}(11+23)\times 2.5 = 42.5$; total 150.6 cm^2 (exact $\frac{753}{5}$).

3.2 (a) $\frac{4}{3}\pi\times 216 = 288\pi$ cm^3. (b) $4\pi\times 36 = 144\pi$ cm^2. (c) $\pi\times 36\times h = 288\pi$ gives $h = 8$ cm.

3.3 (a) Cylinder $\pi\times 4\times 5 = 20\pi$; hemisphere $\frac{2}{3}\pi\times 8 = \frac{16}{3}\pi$; total $\frac{76}{3}\pi$ m^3. (b) Cylinder wall $2\pi\times 2\times 5 = 20\pi$; hemisphere curved $2\pi\times 4 = 8\pi$; total 28π m^2.

Local review guidance: the split drawn or named before formulas; exact π forms kept until the final line; units squared or cubed to match. This is authored classroom guidance.