

4.4 Circle geometry with reasons

Name: _____ Class: _____ Date: _____

Total: 20 marks

KEY WORDS cyclic quadrilateral • chord

Circle geometry with reasons

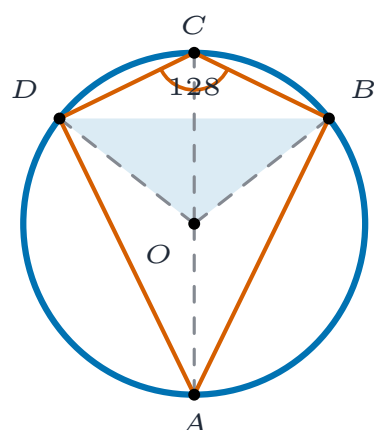
Objective

Use circle vocabulary and chord/tangent properties; at Higher, the cyclic-quadrilateral, centre/circumference and same-segment angle facts and the intersecting chord property – every step with a standard reason (objectives Foundation 4.6–4.7 and Higher 4.6–4.7).

1 Key knowledge and worked examples

The reason bank. Tangent $\perp$ radius at the point of contact; the perpendicular from the centre bisects a chord; angle at centre $= 2 \times$ angle at circumference (same arc); angles in the same segment are equal; the diameter subtends 90° at the circumference; opposite angles of a cyclic quadrilateral sum to 180° ; triangle with two radii is isosceles. Quote the fact in words beside every numerical step.

Intersecting chords. Two chords meeting inside the circle satisfy $AE \times EB = CE \times ED$; the external version does the same from outside. If $AE = 6$, $EB = 8$, $CE = 4$, then $ED = 6 \times 8 \div 4 = 12$.



cyclic quad: $\angle BAD = 52$

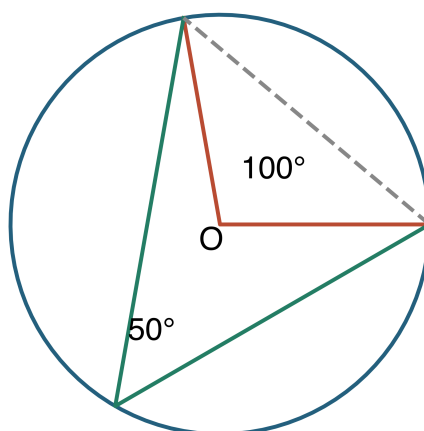
centre = $2 \times$ rim: $\angle BOD = 104$

$OB = OD$ (radii): isosceles

$$\angle OBD = \frac{180 - 104}{2} = 38$$

every numerical step names its reason

The centre angle is twice the circumference angle



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 Write down the angle at the centre when the angle at the circumference on the same arc is 41° . [1]

2.2 $ABCD$ is a cyclic quadrilateral with $\angle ABC = 97^\circ$. Find $\angle ADC$. [2]

2.3 Two chords meet inside a circle: $AE = 6$, $EB = 8$ and $CE = 4$. Find ED . [2]

2.4 A tangent touches the circle at T and O is the centre. State the angle between the tangent and the radius OT . [1]

2.5 PQ is a diameter and R is on the circumference. Find $\angle PRQ$. [1]

3 Exam-style questions

3.1 (*Edexcel 4MA1 1HR June 2024 Q13, adapted.*) A , B , C , D are points on a circle centre O . $\angle BCD = 128^\circ$. Find $\angle OBD$, giving reasons at each stage. [5]

3.2 (*Original.*) A , B , C lie on a circle centre O with $\angle AOB = 130^\circ$. D lies on the major arc AB . (a) Find $\angle ADB$. (b) Find $\angle OAB$. [4]

3.3 (*Original.*) Chords PQ and RS intersect at X inside a circle with $PX = 5$, $QX = 9$, $RX = 6$. (a) Find SX . (b) The perpendicular from O to chord PQ meets it at M ; state PM in terms of PQ and justify. [4]

Solutions

2.1 $2 \times 41 = 82^\circ$.

2.2 $180 - 97 = 83^\circ$ (opposite angles of a cyclic quadrilateral).

2.3 $ED = (6 \times 8)/4 = 12$.

2.4 90° .

2.5 90° (angle in a semicircle).

3.1 Opposite angles of a cyclic quadrilateral: $\angle BAD = 180 - 128 = 52^\circ$. Angle at centre is twice angle at circumference on arc BD : $\angle BOD = 104^\circ$. Triangle OBD is isosceles ($OB = OD$, radii), so $\angle OBD = (180 - 104)/2 = 38^\circ$.

3.2 (a) D lies on the major arc, so $\angle ADB$ subtends the minor arc AB and equals half the angle at the centre: $\angle ADB = 130/2 = 65^\circ$. (b) Triangle OAB is isosceles ($OA = OB$, radii), so $\angle OAB = (180 - 130)/2 = 25^\circ$.

3.3 (a) $PX \times QX = RX \times SX$: $45 = 6 \times SX$, so $SX = 7.5$. (b) $PM = \frac{1}{2}PQ$ – the perpendicular from the centre to a chord bisects it.

Local review guidance: the named theorem before each number; exact angles; reasons in full standard wording, not shorthand. This is authored classroom guidance.