3.6 Mixed Higher practice: sequences, functions and graphs

Name: _____ Class: _____ Date: _____ **Total: 32 marks**

KEY WORDS rate of change

Mixed Higher practice: sequences, functions and graphs

Objective

Combine the topic-3 strands in unfamiliar settings: series sums in context, perpendicular lines, composites and transformations together, intersections with exact answers, and calculus in motion (objectives Higher 3.1–3.4).

1 Key knowledge and worked examples

One line per strand. Arithmetic: $u_n = a + (n-1)d$ and $S_n = \frac{n}{2}[2a + (n-1)d]$. Functions: $\mathrm{fg}(x) = f(g(x))$, g first; invert a one-to-one function by rearranging. Lines: gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$; perpendicular gradient $-1/m$. Graphs: intersections solve $y_2 - y_1 = 0$; transformations $f(x) + a$, $f(x + a)$, $af(x)$, $f(ax)$ act as in the handout. Calculus: $\frac{d}{dx}ax^n = anx^{n-1}$; stationary where $\frac{dy}{dx} = 0$; motion has $v = \frac{ds}{dt}$, $a = \frac{dv}{dt}$.

2 Practice

2.1 Find a rule for the nth term of 6, 11, 16, 21, … [2]

2.2 Find the sum of the first 12 terms of $-3 + 1 + 5 + 9 + \cdots$ [3]

2.3 Find the equation of the line perpendicular to $y = \frac{1}{4}x - 7$ through the point $(5, 2)$. [3]

2.4 For $f(x) = \sqrt{3x - 5}$, write down the smallest value of x allowed in the domain. [2]

2.5 For $y = \frac{1}{4}x^4 - 2x^2$, find the x coordinates of all stationary points. [3]

3 Exam-style questions

3.1 *(Original.)* $f(x) = x^2 - 8x + 20$. (a) Write down the coordinates of the minimum point of $y = f(x)$. (b) The function g is defined by $g(x) = 2f(x-1)$. Find the coordinates of the minimum point of $y = g(x)$. [5]

3.2 *(Original.)* The line $y = 2x - 3$ meets the curve $y = x^2 - x - 6$. (a) Show that the x coordinates of the intersection points satisfy $x^2 - 3x - 3 = 0$. (b) Find the coordinates of both points, giving exact answers. [5]

3.3 *(Original.)* A theatre has 18 seats in row 1, and each row after it has 4 more seats than the row in front. (a) Find the number of seats in row 25. (b) Find the total number of seats in the first 25 rows. [5]

3.4 *(Original.)* A particle moves so that its displacement after t seconds is $s = 2t^3 - 15t^2 + 24t$ metres. (a) Find an expression for v in terms of t. (b) Find the values of t at which the particle is instantaneously at rest. [4]

Solutions

2.1 Differences are 5: $u_n = 6 + 5(n-1) = 5n + 1$.

2.2 $a = -3$, $d = 4$: $S_{12} = 6\,[-6 + 11 \times 4] = 6 \times 38 = 228$.

2.3 Perpendicular gradient -4: $y - 2 = -4(x-5)$, so $y = -4x + 22$.

2.4 $3x - 5 \geq 0$, so $x \geq \frac{5}{3}$.

2.5 $\frac{dy}{dx} = x^3 - 4x = x(x-2)(x+2) = 0$: $x = -2, 0, 2$.

3.1 (a) $f(x) = (x-4)^2 + 4$: minimum $(4, 4)$. (b) $f(x-1)$ shifts right by 1 to $(5, 4)$; multiplying by 2 stretches vertically to $(5, 8)$.

3.2 (a) $x^2 - x - 6 = 2x - 3$ gives $x^2 - 3x - 3 = 0$. (b) $x = \frac{3 \pm \sqrt{9+12}}{2} = \frac{3 \pm \sqrt{21}}{2}$. With $y = 2x - 3 = \pm\sqrt{21}$: $\left(\frac{3+\sqrt{21}}{2}, \sqrt{21}\right)$ and $\left(\frac{3-\sqrt{21}}{2}, -\sqrt{21}\right)$.

3.3 (a) $18 + 24 \times 4 = 114$ seats. (b) $S_{25} = \frac{25}{2}\,[36 + 24 \times 4] = \frac{25}{2} \times 132 = 1650$.

3.4 (a) $v = 6t^2 - 30t + 24$. (b) $6(t-1)(t-4) = 0$: $t = 1$ and $t = 4$ seconds.

Local review guidance: the strand's set-up equation before arithmetic; exact or correctly rounded values with units in context; combined transformations applied in the right order – shift then stretch. This is authored classroom guidance.