3.5 Graphs and transformations

Name: _____ Class: _____ Date: _____ **Total: 27 marks**

KEY WORDS transformation • range

Graphs and transformations

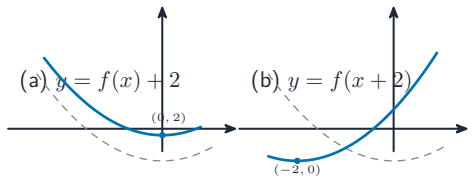
Objective

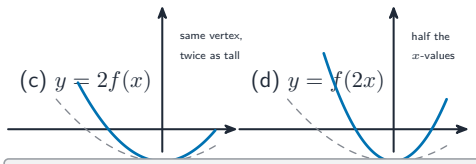
Recognise and sketch cubics, reciprocal curves, sin / cos / tan graphs and the origin-centred circle; apply and read off the four transformations of $y = f(x)$; estimate curve gradients by tangent; solve linear–non-linear intersections (objectives Higher 3.3 A–E).

1 Key knowledge and worked examples

Curve families. Cubics $y = Ax^3 + Bx^2 + Cx + D$ (any signs – $y = x^3 - 4x$ rises–falls–rises); reciprocals $y = A/x$, A/x^2, A/x^3 (two branches, asymptotes, never crossing an axis); $y = \sin x$ and $y = \cos x$ oscillate between -1 and 1 with period 360°; $y = \tan x$ repeats every 180° and is undefined at $90° + 180°n$; a circle centred at the origin has $x^2 + y^2 = r^2$.

Transformations and intersections. $y = f(x) + a$ translates up; $y = f(x + a)$ translates left (inside acts on x, the other way); $y = af(x)$ stretches vertically; $y = f(ax)$ compresses horizontally. A tangent at a point gives the curve's gradient there as rise/run. Intersections of y_1 (linear) and y_2 (non-linear) solve $y_2 - y_1 = 0$: $y = 2x + 1$ with $y = x^2 + 3x - 2$ gives $x^2 + x - 3 = 0$.





the inside forms act on x only, and go the other way: +2 moves left

2 Practice

2.1 The curve $y = \dfrac{6}{x}$ meets the line $x = 2$. Find the coordinates of the point of intersection. [2]

2.2 For $f(x) = x^2$, write down the equation of $y = f(x - 3) + 1$ and the coordinates of its minimum point. [3]

2.3 For $f(x) = x^2 + 4x$, write down the equation of $y = f(2x)$. [2]

2.4 Describe fully the transformation that maps $y = \sin x$ onto $y = \sin(x - 30°)$. [2]

2.5 Write down the quadratic equation whose solutions are the x coordinates of the intersections of $y = 2x + 1$ and $y = x^2 + 3x - 2$. [2]

3 Exam-style questions

3.1 *(Edexcel 4MA1 2HR June 2024 Q22, adapted.)* Find the coordinates of the points where the line $y = 3x - 1$ meets the curve $x^2 + y^2 = y + 11$. [5]

3.2 *(Original.)* $f(x) = x^2 - 6x + 11$. (a) Write down the coordinates of the minimum point of $y = f(x)$. (b) The graph of $y = f(x + k)$ has its minimum point at $(0, 2)$: find k. (c) Write down the coordinates of the minimum point of $y = f(2x)$. [6]

3.3 *(Original.)* The line $y = x + 2$ meets the curve $y = x^3 - 4x$. (a) Show that $x = -2$ is one solution of $x^3 - 5x - 2 = 0$. (b) Find the coordinates of all three points of intersection. [5]

Solutions

2.1 $y = 6/2 = 3$: $(2, 3)$.

2.2 $y = (x-3)^2 + 1$; minimum $(3, 1)$.

2.3 $f(2x) = (2x)^2 + 4(2x) = 4x^2 + 8x$.

2.4 A translation of $30°$ to the right (horizontal shift along x); shape, period and amplitude unchanged.

2.5 $x^2 + 3x - 2 = 2x + 1$, so $x^2 + x - 3 = 0$.

3.1 Substitute $y = 3x - 1$: $x^2 + (3x-1)^2 = (3x-1) + 11$, so $10x^2 - 6x + 1 = 3x + 10$ and $10x^2 - 9x - 9 = 0$. Factorising, $(5x+3)(2x-3) = 0$, so $x = -\frac{3}{5}$ or $x = \frac{3}{2}$. With $y = 3x - 1$: $\left(-\frac{3}{5}, -\frac{14}{5}\right)$ and $\left(\frac{3}{2}, \frac{7}{2}\right)$.

3.2 (a) $f(x) = (x-3)^2 + 2$, so the minimum is $(3, 2)$. (b) $f(x+k)$ shifts the graph k units left, so the minimum sits at $3 - k = 0$: $k = 3$. (c) $f(2x)$ halves the x-values: minimum $(1.5, 2)$.

3.3 (a) $(-2)^3 - 5(-2) - 2 = -8 + 10 - 2 = 0$. ✓ (b) $x^3 - 5x - 2 = (x+2)(x^2 - 2x - 1)$, so $x = 1 \pm \sqrt{2}$ as well. Points: $(-2, 0)$, $\left(1+\sqrt{2},\, 3+\sqrt{2}\right)$ and $\left(1-\sqrt{2},\, 3-\sqrt{2}\right)$.

Local review guidance: substitution of the linear into the non-linear before any expansion; exact coordinates including both points; transformation descriptions name the direction and the variable acted on. This is authored classroom guidance.