

3.4 Differentiation and stationary points

Name: _____ Class: _____ Date: _____

Total: 29 marks

KEY WORDS differentiation • stationary point

Differentiation and stationary points

Objective

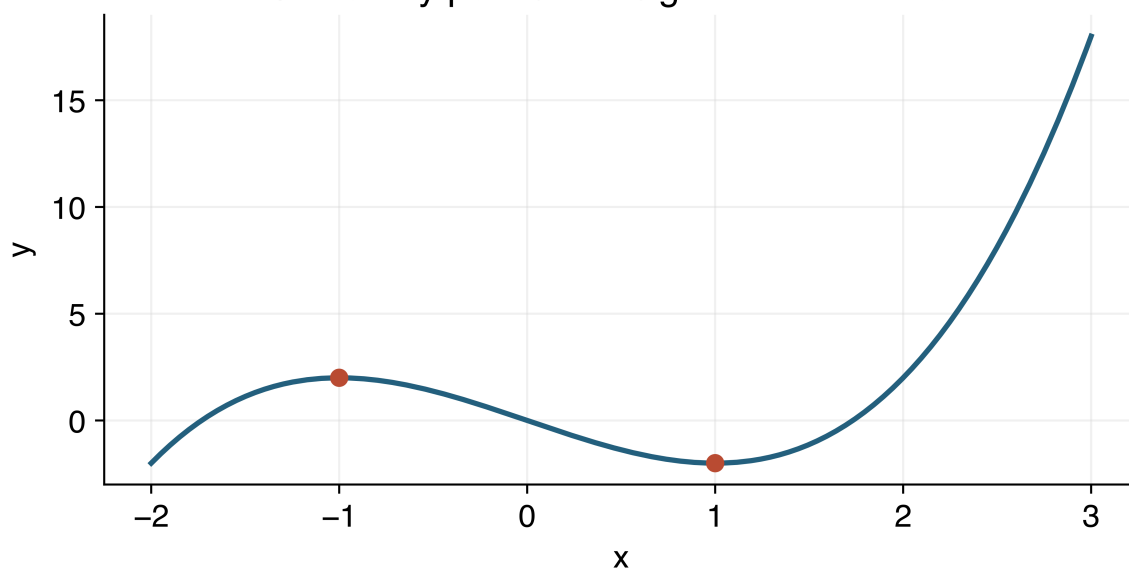
Differentiate integer powers of x , find and classify stationary points from the graph shape, and apply calculus to linear kinematics (objectives Higher 3.4 A–E).

1 Key knowledge and worked examples

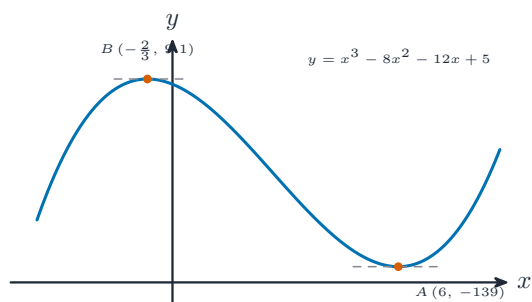
Differentiating integer powers. Term by term, $y = ax^n \Rightarrow \frac{dy}{dx} = anx^{n-1}$: for $y = x^3 - 8x^2 - 12x + 5$, $\frac{dy}{dx} = 3x^2 - 16x - 12 = (3x + 2)(x - 6)$. Stationary points have $\frac{dy}{dx} = 0$. Classify from the graph's shape: a positive cubic rises–falls–rises, so its left stationary point is a maximum and its right one a minimum.

Calculus and motion. With displacement s metres after t seconds: $v = \frac{ds}{dt}$ and $a = \frac{dv}{dt}$. For $s = t^3 - 24t$: $v = 3t^2 - 24$ and $a = 6t$. The particle is instantaneously at rest where $v = 0$ – here $t = \sqrt{8} = 2.83$ s (3 s.f.). Answers carry units when the context does.

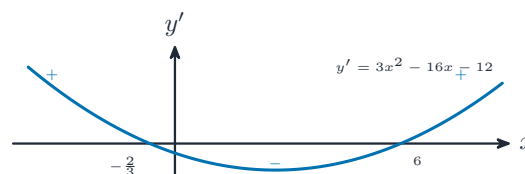
Stationary points and signed accumulation



Original native-lesson illustration; labels belong to its worked example.



stationary where $\frac{dy}{dx} = 0$: $x = -\frac{2}{3}$ or 6



+ to - at B: maximum;
- to + at A: minimum

2 Practice

2.1 Differentiate $y = x^4 - 3x^2 + 7$.

[2]

2.2 For $y = x^3 - 8x^2 - 12x + 5$, find $\frac{dy}{dx}$.

[2]

2.3 Find the coordinates of the stationary point of $y = x^2 - 6x + 5$. [3]

2.4 A particle has $s = t^3 - 24t$ metres after t seconds. Find expressions for its velocity and acceleration. [3]

2.5 Using your answer to 2.4, find the time at which the particle is instantaneously at rest. [2]

3 Exam-style questions

3.1 (*Edexcel 4MA1 1HR June 2024 Q19, adapted.*) A curve C has equation $y = x^3 - 8x^2 - 12x + 5$. C has exactly two stationary points, one at A and one at B , where the x coordinate of A is greater than the x coordinate of B . Find the coordinates of A . Show clear algebraic working. [5]

- 3.2** (*Original.*) The curve $y = x + \frac{4}{x}$ has two stationary points. (a) Find their coordinates. (b) By considering the shape of the curve for $x > 0$, decide which is a minimum. [6]

- 3.3** (*Original.*) A particle moves so that $s = t^3 - 6t^2 + 9t$ metres after t seconds, $0 \leq t \leq 3$. (a) Find an expression for v . (b) Find the times at which the particle is instantaneously at rest. (c) Find the acceleration when $t = 1.5$. [6]

Solutions

2.1 $\frac{dy}{dx} = 4x^3 - 6x$.

2.2 $\frac{dy}{dx} = 3x^2 - 16x - 12$.

2.3 $\frac{dy}{dx} = 2x - 6 = 0$ gives $x = 3$; $y = 9 - 18 + 5 = -4$: $(3, -4)$.

2.4 $v = \frac{ds}{dt} = 3t^2 - 24$; $a = \frac{dv}{dt} = 6t$.

2.5 $3t^2 - 24 = 0$ gives $t^2 = 8$, so $t = 2\sqrt{2} = 2.83$ s (3 s.f.).

3.1 $\frac{dy}{dx} = 3x^2 - 16x - 12 = (3x + 2)(x - 6) = 0$, so $x = -\frac{2}{3}$ or $x = 6$. The larger value is $x = 6$: $y = 216 - 288 - 72 + 5 = -139$, so $A = (6, -139)$.

3.2 (a) $\frac{dy}{dx} = 1 - \frac{4}{x^2} = 0$ gives $x^2 = 4$, so $x = \pm 2$: $(2, 4)$ and $(-2, -4)$. (b) For $x > 0$ the curve $x + 4/x$ is large at both ends and dips between, so $(2, 4)$ is the minimum.

3.3 (a) $v = 3t^2 - 12t + 9$. (b) $3t^2 - 12t + 9 = 3(t - 1)(t - 3) = 0$ gives $t = 1$ and $t = 3$ seconds. (c) $a = 6t - 12$, so $a = -3$ m/s² at $t = 1.5$.

Local review guidance: differentiation with at least two terms right, then the zero gradient equation; coordinates, not just x ; classification argues from shape or gradient signs, not from “it looks like one”. This is authored classroom guidance.