

3.3 Function notation

Name: _____ Class: _____ Date: _____

Total: 22 marks

KEY WORDS composite function • inverse function

Function notation

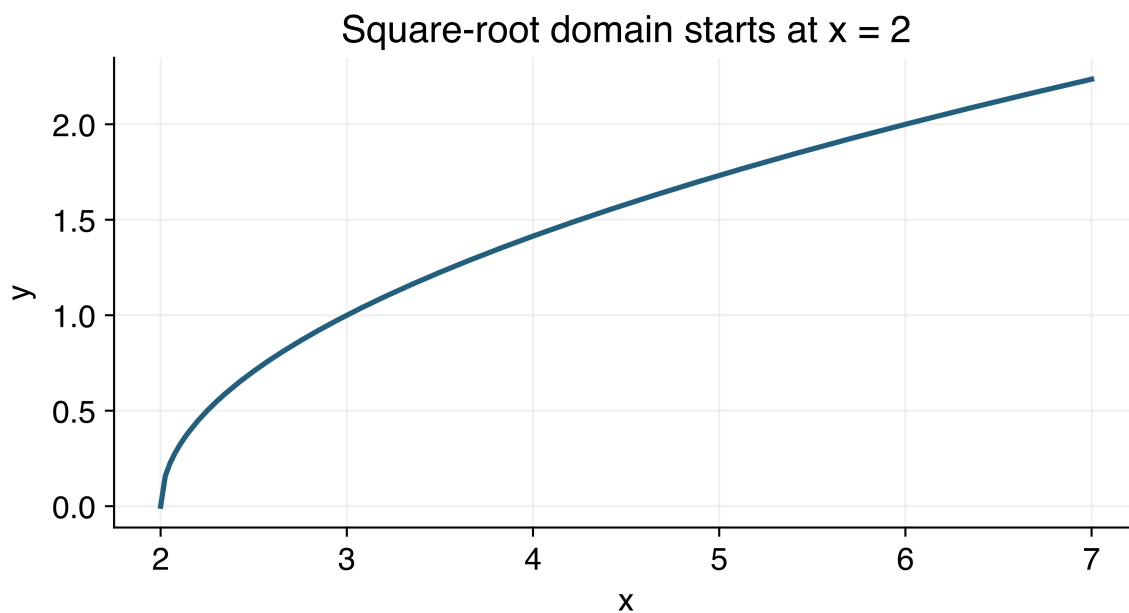
Objective

Use $f(x)$ and mapping notation, state domains with exclusions and ranges, build composite functions fg (apply g first) and find inverse functions on one-to-one domains (objectives Higher 3.2 A–D).

1 Key knowledge and worked examples

Functions, domains, ranges. A function maps each input to exactly one output: $f(x) = \dots$ or $f: x \mapsto \dots$. The domain is the set of allowed inputs – exclude values that break the rule: $f(x) = \frac{1}{x-2}$ excludes $x = 2$; $f(x) = \sqrt{x}$ needs $x \geq 0$. The range is the set of outputs.

Composites and inverses. $fg(x) = f(g(x))$: do g first, and its output must be an allowed input for f . To invert, write $y = f(x)$, rearrange for x , then swap: $f(x) = \sqrt{x-2}$ (domain $x \geq 2$) gives $f^{-1}(x) = x^2 + 2$ on $x \geq 0$. Check $f^{-1}(f(x)) = x$. A parabola needs a restricted domain before it has an inverse.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 For $f(x) = \frac{1}{x - 2}$, write down the value that must be excluded from the domain. [1]

2.2 For $f(x) = 2x + 3$, find $f^{-1}(11)$. [2]

2.3 For $f(x) = \sqrt{x - 2}$ and $g(x) = x + 3$, find $fg(1)$. [2]

2.4 For $f(x) = 3x - 1$, find $f^{-1}(x)$ in terms of x . [2]

2.5 For $f(x) = x^2$ with domain $x \geq 0$, find $f^{-1}(x)$ and state its domain. [2]

3 Exam-style questions

3.1 (*Edexcel 4MA1 2HR June 2024 Q17, adapted.*) $f(x) = \frac{x}{2x-4}$ and $g(x) = 3x + 1$. Given that $fg(k) = 2$, work out the value of k . [3]

3.2 (*Original.*) $g(x) = x^2 - 4x + 7$ for $x \geq 2$. (a) Explain why g has an inverse function for this domain. (b) Find $g^{-1}(x)$ and state its domain. [5]

3.3 (*Original.*) $f(x) = 2x + 1$ and $g(x) = x^2$. (a) Express $gf(x)$ in terms of x . (b) Solve $gf(x) = 27$, giving your answers correct to 3 significant figures. [5]

Solutions

2.1 $x = 2$ (division by zero).

2.2 Solve $2x + 3 = 11$: $x = 4$.

2.3 $g(1) = 4$; $fg(1) = f(4) = \sqrt{2}$.

2.4 $x = \frac{y+1}{3}$, so $f^{-1}(x) = \frac{x+1}{3}$.

2.5 $f^{-1}(x) = \sqrt{x}$ with domain $x \geq 0$ (the range of f).

3.1 $g(k) = 3k + 1$, so $fg(k) = \frac{3k+1}{2(3k+1)-4} = \frac{3k+1}{6k-2} = 2$. Then $3k + 1 = 12k - 4$, so $9k = 5$ and $k = \frac{5}{9}$.

3.2 (a) $g(x) = (x - 2)^2 + 3$ is increasing for $x \geq 2$, so it is one-to-one on the domain and an inverse exists. (b) $y = (x - 2)^2 + 3$ gives $x = 2 + \sqrt{y - 3}$, so $g^{-1}(x) = 2 + \sqrt{x - 3}$ with domain $x \geq 3$ (the range of g).

3.3 (a) $gf(x) = (2x + 1)^2$. (b) $(2x + 1)^2 = 27$ gives $2x + 1 = \pm\sqrt{27}$, so $x = \frac{\sqrt{27}-1}{2} = 2.10$ or $x = \frac{-\sqrt{27}-1}{2} = -3.10$ (3 s.f.).

Local review guidance: correct order for composites – g first inside f ; inverse with its domain, not just the formula; one-to-one arguments via the completed square or a sketch. This is authored classroom guidance.