

3.1 Sequences and series

Name: _____ Class: _____ Date: _____

Total: 25 marks

KEY WORDS common difference • arithmetic sequence

Sequences and series

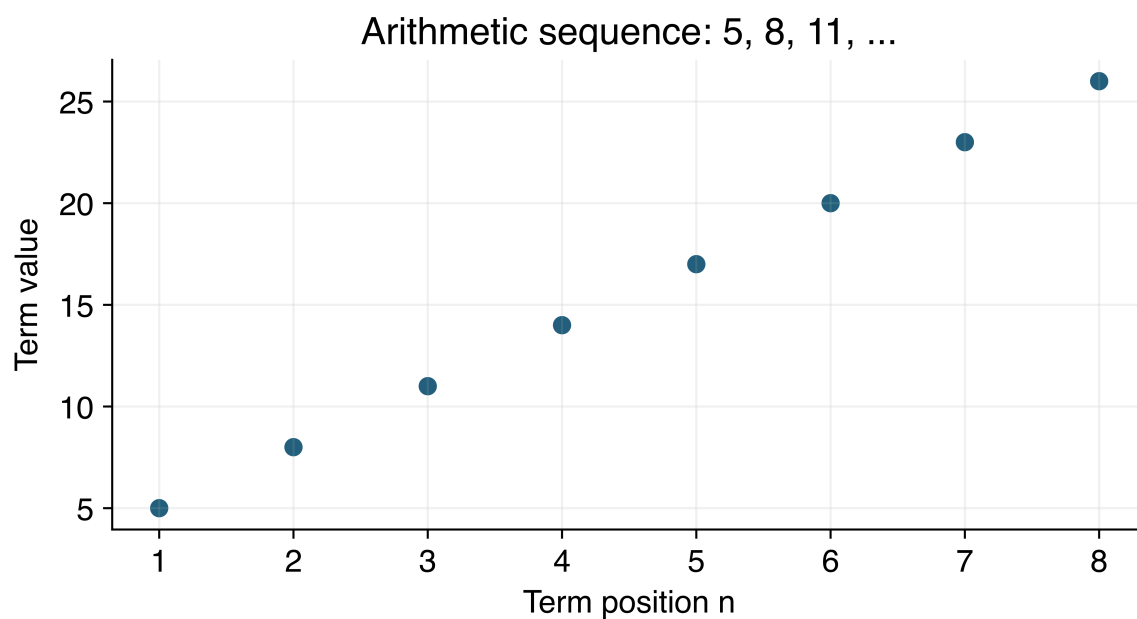
Objective

Generate sequences from term-to-term and position-to-term rules, use linear n th terms, recover a and d from given terms, and sum arithmetic series including algebraic terms (objectives Foundation 3.1 A–C and Higher 3.1 A–C).

1 Key knowledge and worked examples

Two kinds of rule. A term-to-term rule says how to get the next term (add 4: 5, 9, 13, 17, ...; multiply by 2: 1, 2, 4, 8, ...). A position-to-term rule gives a term directly from its position. For an arithmetic sequence, $u_n = a + (n - 1)d$: odds 1, 3, 5, 7, ... have $u_n = 2n - 1$, and $u_n = 4n + 3$ generates 7, 11, 15, ...

Summing an arithmetic series. Pair first with last, second with second-last, ...: each pair sums to $a + u_n$, and there are $n/2$ pairs, so $S_n = \frac{n}{2} [2a + (n - 1)d] = \frac{n}{2} (a + u_n)$. Recovering a and d from two terms uses $u_n = a + (n - 1)d$ twice as simultaneous equations: $u_2 = 7$, $u_5 = 19$ give $3d = 12$, so $d = 4$ and $a = 3$.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 Write down the next two terms of 5, 9, 13, 17, ... and state the term-to-term rule. [2]

2.2 Find a rule for the n th term of 7, 11, 15, 19, ... [2]

2.3 An arithmetic sequence has second term 7 and fifth term 19. Find a and d . [3]

2.4 Find the sum of the first 50 terms of $4 + 7 + 10 + 13 + \dots$ [3]

2.5 Which term of the sequence with n th term $3n + 2$ equals 62? [2]

3 Exam-style questions

3.1 (*Edexcel 4MA1 1HR June 2024 Q23.*) Three consecutive terms of an arithmetic series are $4x - 14$, $x + 2$ and $7x - 9$. Work out the sum of the first 40 terms of the series. [4]

3.2 (*Original.*) The sum of the first three terms of an arithmetic series is 27 and the sum of the first seven terms is 91. Find a and d , and the sum of the first 20 terms. [5]

3.3 (*Original.*) For a series, $S_n = 2n^2 + 3n$. (a) Find the first two terms. (b) Show that the series is arithmetic and find its common difference. [4]

Solutions

2.1 21, 25; the rule is “add 4”.

2.2 Differences are 4, so $u_n = 7 + 4(n - 1) = 4n + 3$.

2.3 $u_5 - u_2 = 3d = 12$, so $d = 4$; $a = 7 - 4 = 3$.

2.4 $S_{50} = \frac{50}{2} [8 + 49 \times 3] = 25 \times 155 = 3875$.

2.5 $3n + 2 = 62$ gives $n = 20$: the 20th term.

3.1 Consecutive terms: $(x + 2) - (4x - 14) = (7x - 9) - (x + 2)$, so $-3x + 16 = 6x - 11$, $x = 3$. The terms are $-2, 5, 12, \dots$, so $a = -2$, $d = 7$ and $S_{40} = 20[-4 + 39 \times 7] = 20 \times 269 = 5380$.

3.2 $S_3 = 3a + 3d = 27$ gives $a + d = 9$; $S_7 = 7a + 21d = 91$ gives $a + 3d = 13$. Subtracting: $2d = 4$, so $d = 2$, $a = 7$. $S_{20} = 10[14 + 19 \times 2] = 10 \times 52 = 520$.

3.3 (a) $u_1 = S_1 = 5$; $u_2 = S_2 - S_1 = (8 + 6) - 5 = 9$. (b) $u_n = S_n - S_{n-1} = 2(2n - 1) + 3 = 4n + 1$, so every term difference is 4: arithmetic with $d = 4$.

Local review guidance: pair-sum or formula structure before substitution; exact values; the common-difference argument for “arithmetic” must reference all terms, not two. This is authored classroom guidance.