2.6 Mixed Higher practice: equations, formulae and identities

Name: _____ Class: _____ Date: _____ **Total: 28 marks**

KEY WORDS algebraic fraction

Mixed Higher practice: equations, formulae and identities

Objective

Combine the topic-2 strands in unfamiliar settings: index equations, subject-twice rearrangements, algebraic-fraction chains, vertex-form optima, inequality regions and context-formed quadratics (objectives Higher 2.1–2.8).

1 Key knowledge and worked examples

One line per strand. Indices: one base, so equate indices. Formulae: same operation both sides; powers need rooting; subject twice means collect and factorise. Fractions: factor, cancel whole factors, invert to divide. Vertex form: minimum at $(-b', c')$ read straight off $a(x+b')^2+c'$. Inequalities: critical values, then the parabola's sign. Pairs: substitute the linear into the quadratic and keep both points.

2 Practice

2.1 Simplify $(x^3)^2 \div x^{-4}$ [2]

2.2 Make g the subject of $T = 2\pi\sqrt{\dfrac{l}{g}}$ [3]

2.3 Simplify $\dfrac{x^2 + 2x - 8}{x^2 - 16}$ [3]

2.4 Write $3x^2 - 12x + 5$ in the form $a(x + b)^2 + c$ [3]

2.5 Solve $4x^2 - 25 \leq 0$ [3]

3 Exam-style questions

3.1 *(Original.)* Given that $\dfrac{2^{3n} \times 2^{n+4}}{2^{2n+1}} = 32$, find the value of n. [3]

3.2 *(Original.)* Write $\dfrac{x+1}{x-2} - \dfrac{3}{x+1}$ as a single fraction in its simplest form. [3]

3.3 *(Original.)* A ball is thrown so that its height after t seconds is $h = 20t - 5t^2$ metres. (a) Write h in the form $a(t+b)^2+c$. (b) State the maximum height and when it occurs. [4]

3.4 *(Original.)* The surface area of a closed cylinder is given by $A = 2\pi r^2 + 2\pi rh$. Make h the subject, then find h when $A = 120\pi$ and $r = 3$. [4]

Solutions

2.1 $x^6 \div x^{-4} = x^{6+4} = x^{10}$.

2.2 $T^2 = \frac{4\pi^2 l}{g}$, so $gT^2 = 4\pi^2 l$ and $g = \dfrac{4\pi^2 l}{T^2}$.

2.3 $\dfrac{(x+4)(x-2)}{(x-4)(x+4)} = \dfrac{x-2}{x-4}$ (for $x \neq \pm 4$).

2.4 $3(x^2 - 4x) + 5 = 3(x-2)^2 - 12 + 5 = 3(x-2)^2 - 7$.

2.5 $(2x-5)(2x+5) \leq 0$: critical values $\pm\frac{5}{2}$; the parabola opens upward, so $-\frac{5}{2} \leq x \leq \frac{5}{2}$.

3.1 Right side: $32 = 2^5$. Indices: $3n + (n+4) - (2n+1) = 2n + 3 = 5$, so $n = 1$.

3.2 $\dfrac{(x+1)^2 - 3(x-2)}{(x-2)(x+1)} = \dfrac{x^2 + 2x + 1 - 3x + 6}{(x-2)(x+1)} = \dfrac{x^2 - x + 7}{(x-2)(x+1)}$ (already simplest).

3.3 (a) $h = -5(t^2 - 4t) = -5(t-2)^2 + 20$. (b) Maximum height 20 m at $t = 2$ seconds.

3.4 $h = \dfrac{A - 2\pi r^2}{2\pi r}$; with $A = 120\pi$, $r = 3$: $h = \frac{120\pi - 18\pi}{6\pi} = \frac{102\pi}{6\pi} = 17$ metres.

Local review guidance: the strand's set-up step before arithmetic; exact simplified forms with stated exclusions; optimum answers read from vertex form, with units in context. This is authored classroom guidance.