2.5 Inequalities and linear–quadratic pairs

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS quadratic inequality

Inequalities and linear–quadratic pairs

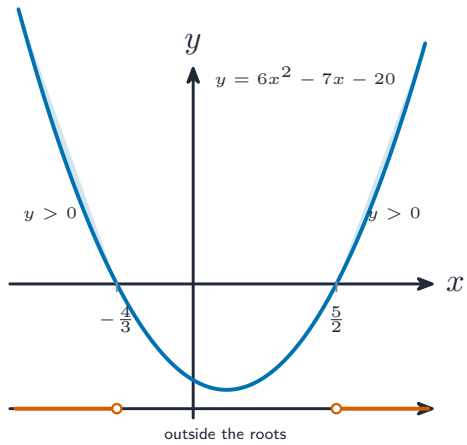
Objective

Use inequality symbols and number-line conventions, solve linear and quadratic inequalities with solution sets on a number line, identify regions defined by linear inequalities, and solve simultaneous pairs where one equation is linear and the other quadratic (objectives Foundation 2.8 and Higher 2.7 D, 2.8).

1 Key knowledge and worked examples

Inequalities. Same moves as equations – but multiplying or dividing by a negative flips the direction: $7 - x \leq 5$ gives $x \geq 2$. Double-ended forms like $1 < x \leq 5$ carry open (excluded) and closed (included) circles on the number line. Quadratic inequalities: find the critical values, then use the parabola's sign – a positive quadratic is greater than zero outside its roots.

Linear–quadratic pairs. Substitute the linear equation into the quadratic, solve the resulting quadratic, then substitute back for the other variable. For $y = 11x - 2$ with $y = 5x^2$: $5x^2 - 11x + 2 = (5x - 1)(x - 2) = 0$, giving $\left(\frac{1}{5}, 0.2\right)$ and $(2, 20)$. Both points are kept unless the context removes one.



positive quadratic: $>$ holds
outside the critical values

2 Practice

2.1 Solve $3x - 2 < 10$ and show the solution set on a number line. [2]

2.2 Solve $7 - x \leq 5$ [2]

2.3 Solve $3 < x + 2 \leq 5$ [2]

2.4 Write down the critical values of $6x^2 - 7x - 20 = 0$ [3]

2.5 Solve $y = 3x - 1$ and $y = x + 3$ simultaneously. [2]

3 Exam-style questions

3.1 *(Edexcel 4MA1 2HR June 2024 Q20, adapted.)* Solve the inequality $6x^2 - 7x - 20 > 0$ [4]

3.2 *(Edexcel-style, specification example.)* Solve the simultaneous equations $y = 2x - 11$ and $x^2 + y^2 = 25$ [4]

3.3 *(Original.)* (a) Solve $x^2 + 3x + 2 > 0$. (b) Shade, on sketch axes, the region satisfying $x \geq 0$, $y \geq 1$ and $x + y \leq 5$, and write down the coordinates of the region's three vertices. [6]

Solutions

2.1 $3x < 12$, so $x < 4$: open circle at 4, arrow left.

2.2 $-x \leq -2$, and dividing by -1 flips the sign: $x \geq 2$.

2.3 Subtract 2 throughout: $1 < x \leq 3$.

2.4 $(3x+4)(2x-5) = 0$: $x = -\frac{4}{3}$ or $x = \frac{5}{2}$.

2.5 $3x - 1 = x + 3$, so $x = 2$ and $y = 5$.

3.1 $(3x+4)(2x-5) > 0$: critical values $-\frac{4}{3}$ and $\frac{5}{2}$. The parabola opens upward, so it is positive outside the roots: $x < -\frac{4}{3}$ or $x > \frac{5}{2}$.

3.2 Substitute: $x^2 + (2x-11)^2 = 25$ gives $x^2 + 4x^2 - 44x + 121 = 25$, so $5x^2 - 44x + 96 = 0$ and $x = \frac{44 \pm \sqrt{1936-1920}}{10} = \frac{44 \pm 4}{10} = 4.8$ or 4. With $y = 2x - 11$: $(4.8, -1.4)$ and $(4, -3)$.

3.3 (a) $(x+1)(x+2) > 0$: positive outside the roots, so $x < -2$ or $x > -1$. (b) The boundaries $x = 0$, $y = 1$ and $x + y = 5$ enclose a triangle with vertices $(0, 1)$, $(0, 5)$ and $(4, 1)$; shade inside it.

Local review guidance: critical values before any inequality direction decision; strict vs non-strict notation matches the original sign; linear–quadratic pairs keep both solutions with both coordinates. This is authored classroom guidance.