2.4 Completing the square and solving quadratics

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS completing the square

Completing the square and solving quadratics

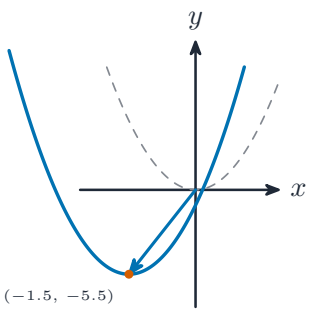
Objective

Complete the square for any quadratic, read the vertex form, and solve quadratic equations by factorising, by the quadratic formula, or by completing the square, including equations formed from context (objectives Higher 2.2 D and 2.7 A–C).

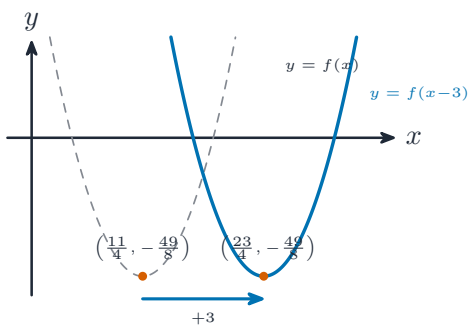
1 Key knowledge and worked examples

Completing the square. Half the x-coefficient gives the shift, inside the bracket, sign flipped: $2x^2 + 6x - 1 = 2(x + 1.5)^2 - 5.5$. The vertex form exposes the minimum directly – the graph is $y = 2x^2$ translated by $(-1.5, -5.5)$.

Three solving routes. Factorise first ($x^2 + x - 30 = (x + 6)(x - 5)$); if it will not factorise, the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ always works ($2x^2 + 6x - 1 = 0$ gives $x = \frac{-3 \pm \sqrt{11}}{2}$); completing the square solves and gives the vertex at once. Keep exact answers; reject roots the context forbids.



dashed $y = 2x^2$, solid $y = 2(x+1.5)^2 - 5.5$; the vertex moves to $(-1.5, -5.5)$



inside the bracket acts on x, the other way

2 Practice

2.1 Write $x^2 + 6x + 5$ in the form $(x + b)^2 + c$ [2]

2.2 Hence state the coordinates of the minimum point of $y = x^2 + 6x + 5$ [1]

2.3 Solve $x^2 + x - 30 = 0$ by factorisation [2]

2.4 Solve $2x^2 + 6x - 1 = 0$, giving exact answers [3]

2.5 Write $2x^2 - 11x + 9$ in the form $a(x + b)^2 + c$ [3]

3 Exam-style questions

3.1 *(Edexcel 4MA1 1HR June 2024 Q20, adapted.)* (a) Write $2x^2 - 11x + 9$ in the form $a(x+b)^2+c$. (b) The graph of $y = f(x-3)$, where $f(x) = 2x^2 - 11x + 9$, has a minimum point. Find the coordinates of this minimum point. [5]

3.2 *(Original.)* A rectangular lawn has length 3 m more than its width. Its area is 70 m^2. Form and solve a quadratic equation to find the width and length. [4]

3.3 *(Original.)* Solve $3x^2 = 4x + 2$ giving answers correct to 2 decimal places. [3]

Solutions

2.1 $x^2 + 6x + 5 = (x+3)^2 - 9 + 5 = (x+3)^2 - 4$.

2.2 Minimum $(-3, -4)$.

2.3 $(x+6)(x-5) = 0$: $x = -6$ or 5.

2.4 $x = \dfrac{-6 \pm \sqrt{36+8}}{4} = \dfrac{-6 \pm \sqrt{44}}{4} = \dfrac{-3 \pm \sqrt{11}}{2}$.

2.5 $2x^2 - 11x + 9 = 2\left(x - \frac{11}{4}\right)^2 - \frac{121}{8} + 9 = 2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8}$.

3.1 (a) As 2.5: $2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8}$. (b) f has its minimum at $\left(\frac{11}{4}, -\frac{49}{8}\right)$; $f(x-3)$ shifts 3 right, so the minimum is $\left(\frac{23}{4}, -\frac{49}{8}\right) = (5.75, -6.125)$.

3.2 Width w: $w(w+3) = 70$, so $w^2 + 3w - 70 = 0 = (w+10)(w-7)$. Width 7 m (positive root), length 10 m. Check: $7 \times 10 = 70$. ✓

3.3 $3x^2 - 4x - 2 = 0$: $x = \dfrac{4 \pm \sqrt{16+24}}{6} = \dfrac{4 \pm \sqrt{40}}{6} = 1.72$ or -0.39 (2 d.p.).

Local review guidance: half-the-coefficient structure with the correcting constant; exact values unless decimals are asked; context rejects the impossible root with a reason. This is authored classroom guidance.