

2.3 Linear equations, proportion and simultaneous pairs

Name: _____ Class: _____ Date: _____

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KEY WORDS proportion • simultaneous linear equations

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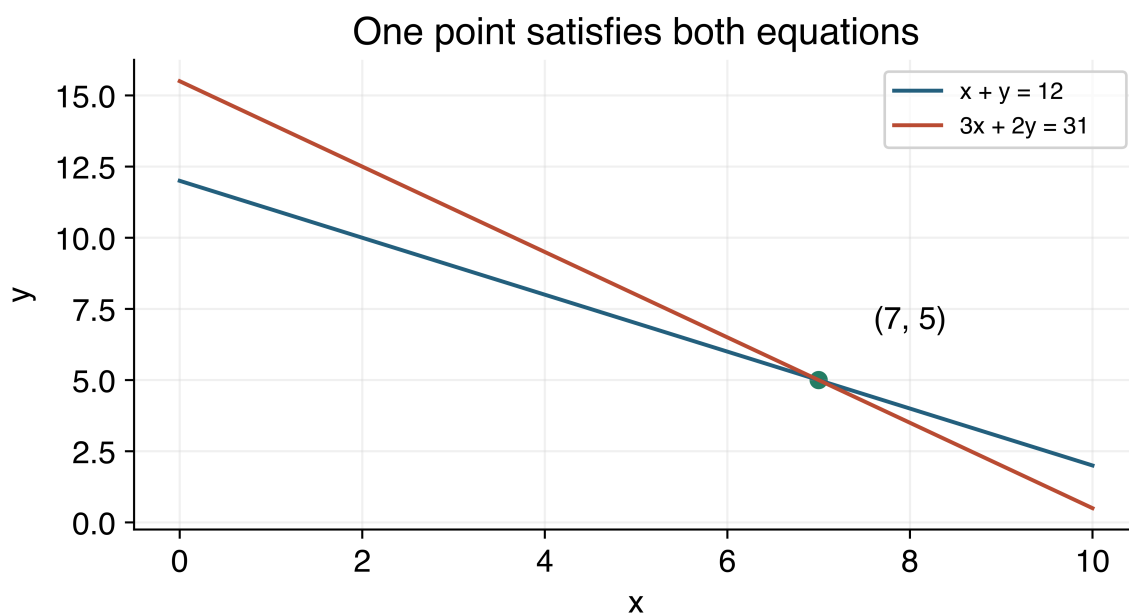
Objective

Solve linear equations from data, set up direct and inverse proportion, and solve simultaneous linear pairs exactly by elimination, interpreting them as intersecting lines (objectives Foundation 2.4, 2.6 and Higher 2.5, 2.6).

1 Key knowledge and worked examples

Linear equations. Expand, collect, divide – the unknown may sit on either side or both: $7(x + 3) = 5x + 8$ gives $2x = -13$, so $x = -6.5$. Check by substitution; reject answers the context forbids (a count cannot be negative or fractional).

Proportion and pairs. $y \propto x^n$ means $y = kx^n$: find k from one data pair. For simultaneous pairs, scale one equation so one unknown matches, then add or subtract: $2x + 3y = 17$ with $3x - 5y = 35$ triples/doubles to $6x + 9y = 51$ and $6x - 10y = 70$; subtracting, $19y = -19$, so $y = -1$, $x = 10$. Each equation is a line; the solution is their intersection.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 Solve $7(x + 3) = 5x + 8$ [2]

2.2 Solve $\frac{4}{5}x + 3 = \frac{2}{3}$ [3]

2.3 Solve the pair $x + y = 14$, $x - y = 2$ [2]

2.4 Solve the pair $2a + 5b = 12$, $3a + b = 5$ [3]

2.5 y is directly proportional to x^3 ; $y = 24$ when $x = 2$. Find y when $x = 3$. [3]

2.6 p is inversely proportional to x^2 ; $p = 8$ when $x = 2$. Find p when $x = 4$. [3]

3 Exam-style questions

3.1 (*Edexcel-style, specification example.*) Solve the simultaneous equations $2x + 3y = 17$ and $3x - 5y = 35$. [3]

3.2 (*Original.*) A cinema ticket for an adult costs $\$(a)$ and for a child $\$(c)$. Two adults and three children pay $\$34$; one adult and four children pay $\$27$. (a) Form two equations. (b) Find the cost of each ticket. [4]

3.3 (*Original.*) y is inversely proportional to x . When $x = 6$, $y = 5$. (a) Find the equation linking x and y . (b) Find x when $y = 2$. (c) Sketch, or describe, the graph of y against x . [4]

Solutions

2.1 $7x + 21 = 5x + 8$, so $2x = -13$ and $x = -6.5$.

2.2 Multiply by 15: $12x + 45 = 10$, so $x = -\frac{35}{12}$.

2.3 Adding: $2x = 16$, $x = 8$; then $y = 6$.

2.4 From the second, $b = 5 - 3a$; substituting: $2a + 25 - 15a = 12$, so $-13a = -13$, $a = 1$, $b = 2$.

2.5 $y = kx^3$: $24 = 8k$ gives $k = 3$; at $x = 3$, $y = 3 \times 27 = 81$.

2.6 $p = k/x^2$: $8 = k/4$ gives $k = 32$; at $x = 4$, $p = 32/16 = 2$.

3.1 Triple the first, double the second: $6x + 9y = 51$ and $6x - 10y = 70$. Subtracting: $19y = -19$, so $y = -1$ and $x = \frac{17 - 3(-1)}{2} = 10$. Check: $3(10) - 5(-1) = 35$. ✓

3.2 (a) $2a + 3c = 34$ and $a + 4c = 27$. (b) From the second, $a = 27 - 4c$; substituting: $54 - 8c + 3c = 34$, so $5c = 20$, $c = 4$ and $a = 27 - 16 = 11$: adult \$11, child \$4. Check: $2(11) + 3(4) = 34$. ✓

3.3 (a) $y = k/x$: $5 = k/6$ gives $k = 30$, so $y = 30/x$. (b) $2 = 30/x$ gives $x = 15$. (c) A hyperbola in the first (and third) quadrant, never touching either axis.

Local review guidance: elimination or substitution with a stated multiplier; exact answers checked in the unused equation; proportion models stated as $y = kx^n$ before substitution. This is authored classroom guidance.