

2.2 Quadratic algebra: factorising, fractions and proofs

Name: _____ Class: _____ Date: _____

Total: 21 marks

KEY WORDS quadratic expression • proof

Quadratic algebra: factorising, fractions and proofs

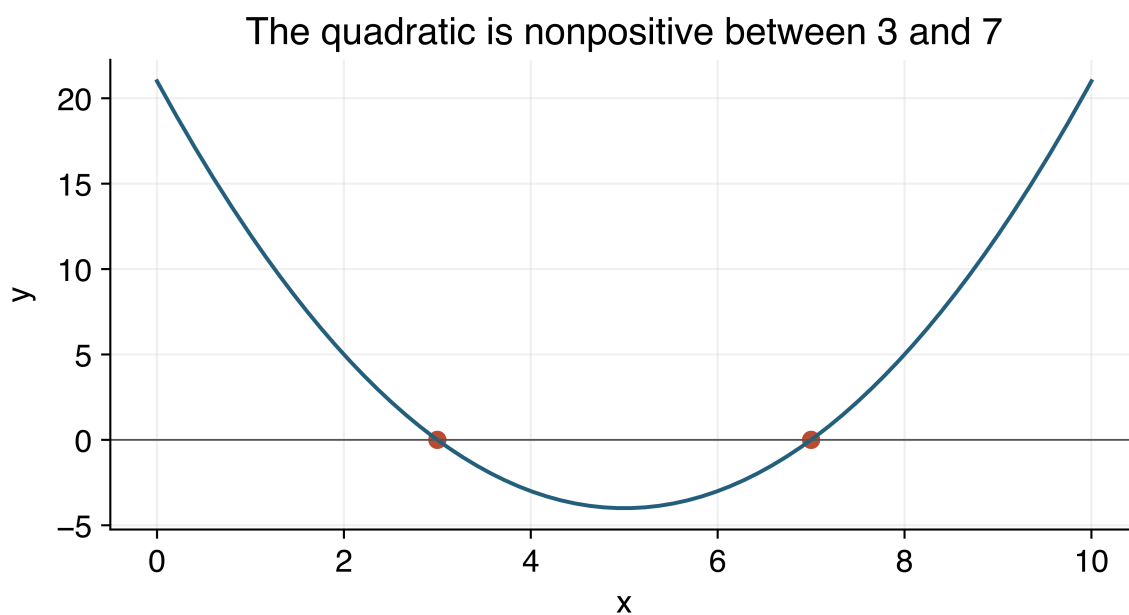
Objective

Factorise quadratic expressions (any a), manipulate algebraic fractions including division, and construct algebraic proofs such as recurring-decimal conversions (objectives Foundation 2.7 A and Higher 2.2 B, C, E).

1 Key knowledge and worked examples

Factorising. $x^2 + bx + c$: find a pair summing to b , multiplying to c . Any quadratic: $6x^2 - 5x - 6 = (3x + 2)(2x - 3)$ – check by expanding. Difference of two squares: $4x^2 - 9 = (2x - 3)(2x + 3)$. Always take out common factors first.

Fractions and proofs. Factor numerator and denominator, then cancel whole factors: $\frac{x^2-4}{x^2+5x+6} = \frac{(x-2)(x+2)}{(x+2)(x+3)} = \frac{x-2}{x+3}$. To divide, invert and multiply. A recurring decimal is an algebra proof: let x equal the decimal, multiply by the power of 10 that shifts one full block, subtract, simplify.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 Factorise $x^2 + x - 30$ [2]

2.2 Factorise $6x^2 - 5x - 6$ [2]

2.3 Factorise fully $4x^2 - 9$ [2]

2.4 Simplify $\frac{x^2 - 4}{x^2 + 5x + 6}$ [3]

2.5 Express $\frac{3}{x+1} + \frac{2}{x-2}$ as a single fraction [3]

3 Exam-style questions

3.1 (*Edexcel 4MA1 2HR June 2024 Q26, adapted.*) Write $4 - (3x - 5) \div \frac{3x^2 + x - 10}{4x - 1}$ as a single fraction in its simplest form. [4]

3.2 (*Edexcel 4MA1 2HR June 2024 Q18, adapted.*) Using algebra, convert the recurring decimal $0.306306\dots$ to a fraction in its simplest form. [2]

3.3 (*Original.*) Prove that $(2n + 1)^2 - (2n - 1)^2$ is a multiple of 8 for every integer n . [3]

Solutions

2.1 $(x + 6)(x - 5)$.

2.2 $(3x + 2)(2x - 3)$.

2.3 $(2x - 3)(2x + 3)$.

2.4 $\frac{(x - 2)(x + 2)}{(x + 2)(x + 3)} = \frac{x - 2}{x + 3}$ (for $x \neq -2, -3$).

2.5 $\frac{3(x - 2) + 2(x + 1)}{(x + 1)(x - 2)} = \frac{5x - 4}{(x + 1)(x - 2)}$.

3.1 Factor: $3x^2 + x - 10 = (3x - 5)(x + 2)$. Dividing inverts: $4 - \frac{(3x - 5)(4x - 1)}{(3x - 5)(x + 2)} =$
 $4 - \frac{4x - 1}{x + 2} = \frac{4(x + 2) - (4x - 1)}{x + 2} = \frac{9}{x + 2}$.

3.2 Let $x = 0.306306\dots$. Then $1000x = 306.306\dots$, and subtracting, $999x = 306$, so $x = \frac{306}{999} = \frac{34}{111}$.

3.3 $(2n + 1)^2 - (2n - 1)^2 = (4n^2 + 4n + 1) - (4n^2 - 4n + 1) = 8n$, which is a multiple of 8 for every integer n .

Local review guidance: factor before cancelling, and cancel only whole factors; simplest-form single fraction; proofs are algebraic – a numeric example is not a proof. This is authored classroom guidance.