Exercise walk-through

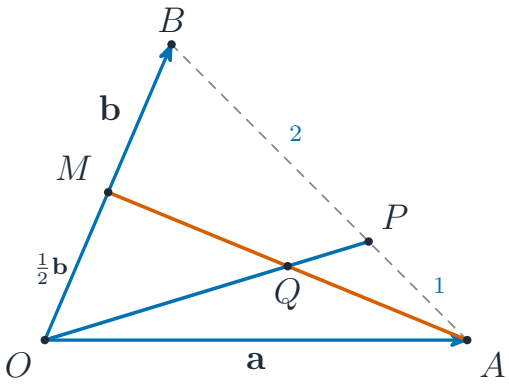
5.5
Mixed practice: vectors and transformations

Pearson Edexcel · International GCSE · Mathematics A

Vector geometry proofs

- To show two lines are **parallel**, show that one vector is a **scalar multiple** of the other.
- To show three points lie on one straight line, show that two of the vectors joining them are parallel and share a point.
- To find an unknown ratio, write the same vector in two different ways and compare the **coefficients** of $\mathbf{a}$ and of $\mathbf{b}$.

Figure: Vector geometry proofs



write $\overrightarrow{OQ}$ along OP and along AM,
then equate: $k = \frac{3}{4}$, $\lambda = \frac{1}{2}$

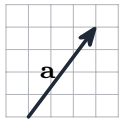
- Write OQ two ways, along OP and along AM
- Equate coefficients: k = 3/4, so OQ : OP = 3 : 4

Transformations: rotation, reflection, translation

- A **reflection** flips the shape in a **mirror line**, such as $x = 1$, $y = 2$ or $y = x$. Each point and its **image** are the same distance from the line.
- A **rotation** turns the shape about a centre through an angle, **clockwise** or **anticlockwise**.
- A **translation** slides the shape without turning it. Describe it with a **column vector**: $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ means 3 right and 2 down.

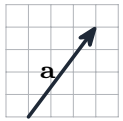
Exercise 2.1

2.1 $\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Find $|\mathbf{a}|$ and $|2\mathbf{a}|$. [2]



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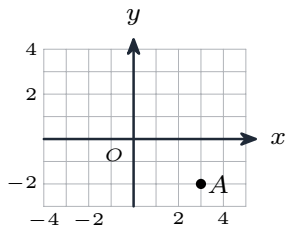


2.1

- $|\mathbf{a}| = \mathbf{5}$
- $|2\mathbf{a}| = \mathbf{10}$

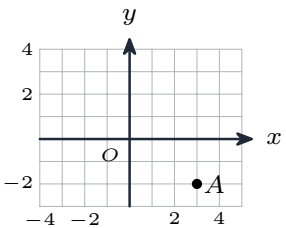
Exercise 2.2

2.2 The point $A(3,\ -2)$ is translated by $\begin{pmatrix} -4 \\ 5 \end{pmatrix}$ and then reflected in the y-axis. Find the final point. [2]



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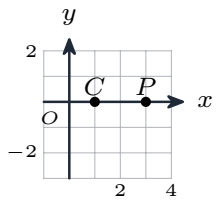


2.2

- after the translation: $(-1,\ 3)$
- after the reflection: $(1,\ 3)$

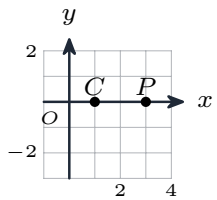
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2.3 The point $P(3,\ 0)$ is rotated $90°$ clockwise about the point $C(1,\ 0)$. Find the coordinates of the image. [2]



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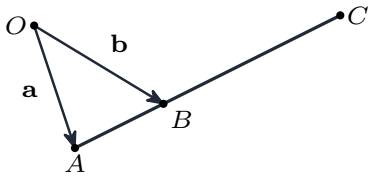


2.3

- vector from C is $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$, which turns to $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$
- image $(1,\ -2)$

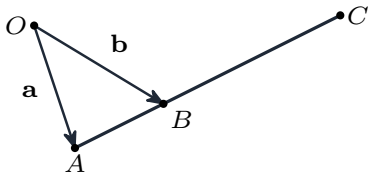
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3.1 $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The line AB is extended to C so that $AC = 3AB$. Find $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. Simplify your answer. [3]



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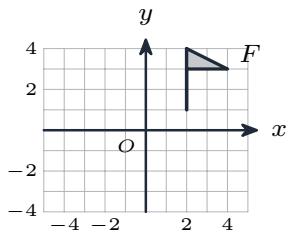


3.1

- $\overrightarrow{AC} = 3(\mathbf{b} - \mathbf{a})$
- $\overrightarrow{OC} = \mathbf{a} + 3\mathbf{b} - 3\mathbf{a}$
- $= 3\mathbf{b} - 2\mathbf{a}$

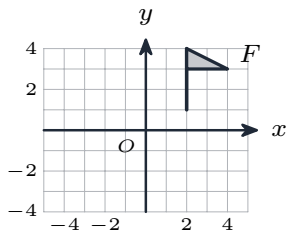
Exercise 3.2

3.2 Flag F is reflected in the x-axis and then the image is reflected in the y-axis. Describe fully the single transformation that maps F onto the final image. [3]



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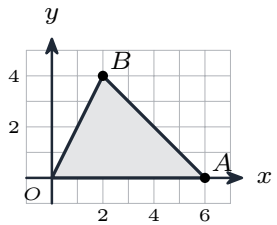
3.2

- rotation
- 180°
- about (0, 0)

Exercise 3.3 (a)

3.3 O is the origin. $\overrightarrow{OA} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

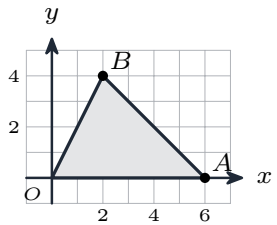
(a) Find $\overrightarrow{AB}$ and $|\overrightarrow{AB}|$. [3]



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(a) Find $\overrightarrow{AB}$ and $|\overrightarrow{AB}|$. [3]



(a) $\begin{pmatrix} -4 \\ 4 \end{pmatrix}$; $\sqrt{32} = \mathbf{5.66}$

Exercise 3.3 (b)

3.3 O is the origin. $\overrightarrow{OA} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

(b) $\overrightarrow{OP} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$. Find $\overrightarrow{OP}$ and say where P is. [3]

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(b) $\overrightarrow{OP} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$. Find $\overrightarrow{OP}$ and say where P is. [3]

(b) $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$; $\overrightarrow{AP} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} = \frac{1}{2}\overrightarrow{AB}$; P is the **midpoint** of AB

Exercise 3.3 (c)

3.3 O is the origin. $\overrightarrow{OA} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

(c) Triangle OAB is enlarged by scale factor $\frac{1}{2}$, centre O, to give $OA'B'$. Find $\overrightarrow{OA'}$ and $\overrightarrow{OB'}$. Show that $A'B'$ is parallel to AB and half as long. [4]

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(c) $\overrightarrow{OA'} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$; $\overrightarrow{OB'} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$; $\overrightarrow{A'B'} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} = \frac{1}{2}\overrightarrow{AB}$

Exercise 3.3 (d)

3.3 O is the origin. $\overrightarrow{OA} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

(d) Triangle OAB is rotated $90°$ anticlockwise about O to give $OA''B''$. Find the coordinates of A'' and B'', find $\overrightarrow{A''B''}$, and explain why $|\overrightarrow{A''B''}| = |\overrightarrow{AB}|$. [5]

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(d) $A''(0,\ 6)$; $B''(-4,\ 2)$; $\overrightarrow{A''B''} = \begin{pmatrix} -4 \\ -4 \end{pmatrix}$; a rotation keeps all lengths the same, so both are $\sqrt{32}$