Exercise walk-through

5.2
Transformations

Pearson Edexcel · International GCSE · Mathematics A

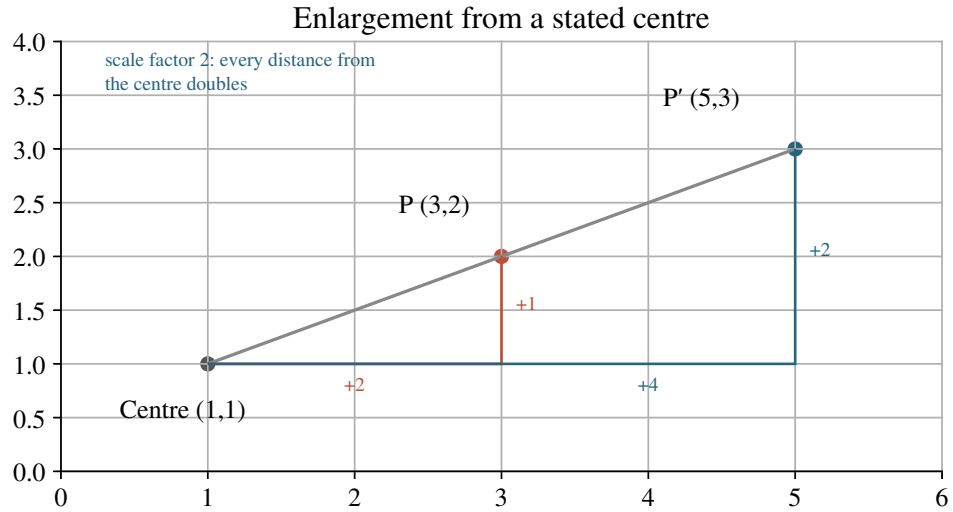
Vector geometry proofs

- To show two lines are **parallel**, show that one vector is a **scalar multiple** of the other.
- To show three points lie on one straight line, show that two of the vectors joining them are parallel and share a point.
- To find an unknown ratio, write the same vector in two different ways and compare the **coefficients** of $\mathbf{a}$ and of $\mathbf{b}$.

Transformations: rotation, reflection, translation

- A **reflection** flips the shape in a **mirror line**, such as $x = 1$, $y = 2$ or $y = x$. Each point and its **image** are the same distance from the line.
- A **rotation** turns the shape about a centre through an angle, **clockwise** or **anticlockwise**.
- A **translation** slides the shape without turning it. Describe it with a **column vector**: $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ means 3 right and 2 down.

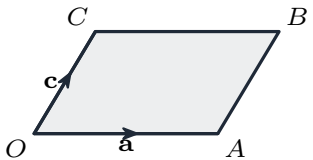
Figure: Enlargement and describing transformations



- **Scale factor** 2 from (1, 1): P(3, 2) $\rightarrow$ (5, 3)
- The **image** is twice as far from the centre

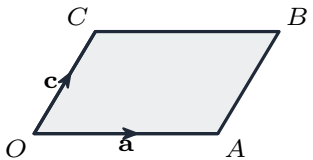
Exercise 2.1

2.1 $OABC$ is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. Find $\overrightarrow{OB}$ and $\overrightarrow{AC}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. [2]



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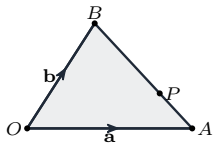


2.1

- $\overrightarrow{OB} = \mathbf{a} + \mathbf{c}$
- $\overrightarrow{AC} = \mathbf{c} - \mathbf{a}$

Exercises 2.2·2.3

2.2 $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. P is on AB with $AP : PB = 1 : 2$. Find $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. Simplify your answer. [2]

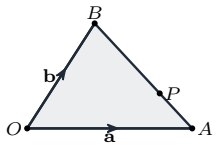


2.3 Show that the vectors $3\mathbf{a} - 6\mathbf{b}$ and $\mathbf{a} - 2\mathbf{b}$ are parallel. [2]

$3\mathbf{a} - 6\mathbf{b}$ $\mathbf{a} - 2\mathbf{b}$

Exercises 2.2·2.3

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2.2

- $\mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a})$
- $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

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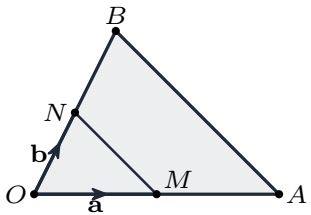
$3\mathbf{a} - 6\mathbf{b}$ $\mathbf{a} - 2\mathbf{b}$

2.3

- $3\mathbf{a} - 6\mathbf{b} = 3(\mathbf{a} - 2\mathbf{b})$
- one is 3 times the other, so they are parallel

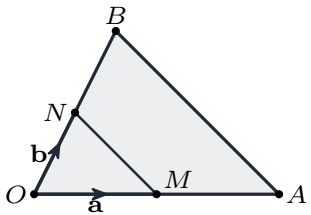
Exercise 3.1

3.1 In triangle OAB, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. M is the midpoint of OA and N is the midpoint of OB. Prove that MN is parallel to AB and half as long. [3]



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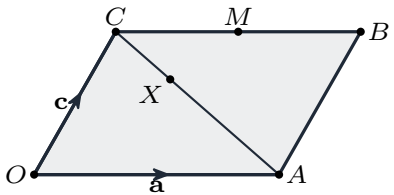
3.1

- $\overrightarrow{MN} = -\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$
- $= \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}\overrightarrow{AB}$
- so MN is parallel to AB and half its length

Exercise 3.2 (a)

3.2 $OABC$ is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. M is the midpoint of BC. X is the point on AC with $AX : XC = 2 : 1$.

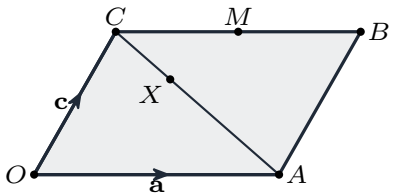
(a) Find $\overrightarrow{OB}$ and $\overrightarrow{AC}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. [2]



Exercise 3.2 (a)

3.2 $OABC$ is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. M is the midpoint of BC. X is the point on AC with $AX : XC = 2 : 1$.

(a) Find $\overrightarrow{OB}$ and $\overrightarrow{AC}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. [2]



(a) $\overrightarrow{OB} = \mathbf{a} + \mathbf{c}$; $\overrightarrow{AC} = \mathbf{c} - \mathbf{a}$

Exercise 3.2 (b)

3.2 $OABC$ is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. M is the midpoint of BC. X is the point on AC with $AX : XC = 2 : 1$.

(b) Find $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. [3]

Exercise 3.2 (b)

3.2 $OABC$ is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. M is the midpoint of BC. X is the point on AC with $AX : XC = 2 : 1$.

(b) Find $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. [3]

(b) $\overrightarrow{OM} = \overrightarrow{OC} + \overrightarrow{CM}$; $\overrightarrow{CM} = \frac{1}{2}\mathbf{a}$; $\overrightarrow{OM} = \mathbf{c} + \frac{1}{2}\mathbf{a}$

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Exercise 3.2 (c)

3.2 $OABC$ is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. M is the midpoint of BC. X is the point on AC with $AX : XC = 2 : 1$.

(c) Find $\overrightarrow{OX}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. Simplify your answer. [4]

Exercise 3.2 (c)

3.2 $OABC$ is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. M is the midpoint of BC. X is the point on AC with $AX : XC = 2 : 1$.

(c) Find $\overrightarrow{OX}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. Simplify your answer. [4]

(c) $\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$; $\overrightarrow{AX} = \frac{2}{3}(\mathbf{c} - \mathbf{a})$; $\mathbf{a} + \frac{2}{3}\mathbf{c} - \frac{2}{3}\mathbf{a} = \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{c}$

Exercise 3.2 (d)

3.2 $OABC$ is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. M is the midpoint of BC. X is the point on AC with $AX : XC = 2 : 1$.

(d) Prove that O, X and M lie on a straight line, and find the ratio $OX : XM$. [5]

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3.2 $OABC$ is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. M is the midpoint of BC. X is the point on AC with $AX : XC = 2 : 1$.

(d) Prove that O, X and M lie on a straight line, and find the ratio $OX : XM$. [5]

(d) $\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{c} = \frac{2}{3}(\frac{1}{2}\mathbf{a} + \mathbf{c})$; so $\overrightarrow{OX} = \frac{2}{3}\overrightarrow{OM}$; the vectors are parallel and share the point O, so O, X, M lie on a straight line; $OX : XM = \mathbf{2} : \mathbf{1}$