Exercise walk-through

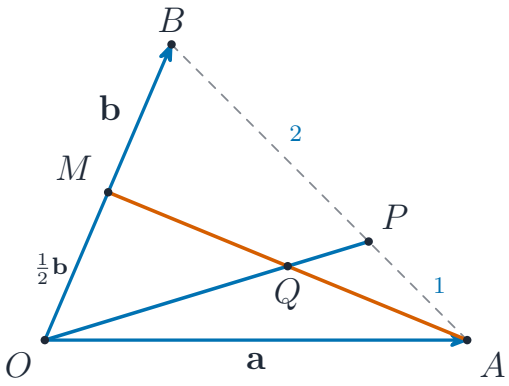
5.1 Vectors

Pearson Edexcel · International GCSE · Mathematics A

Vector geometry proofs

- To show two lines are **parallel**, show that one vector is a **scalar multiple** of the other.
- To show three points lie on one straight line, show that two of the vectors joining them are parallel and share a point.
- To find an unknown ratio, write the same vector in two different ways and compare the **coefficients** of $\mathbf{a}$ and of $\mathbf{b}$.

Figure: Vector geometry proofs



write $\overrightarrow{OQ}$ along OP and along AM,
then equate: $k = \frac{3}{4}$, $\lambda = \frac{1}{2}$

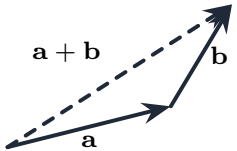
- Write OQ two ways, along OP and along AM
- Equate coefficients: k = 3/4, so OQ : OP = 3 : 4

Transformations: rotation, reflection, translation

- A **reflection** flips the shape in a **mirror line**, such as $x = 1$, $y = 2$ or $y = x$. Each point and its **image** are the same distance from the line.
- A **rotation** turns the shape about a centre through an angle, **clockwise** or **anticlockwise**.
- A **translation** slides the shape without turning it. Describe it with a **column vector**: $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ means 3 right and 2 down.

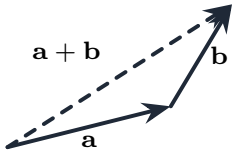
Exercise 2.1

2.1 Write $\mathbf{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$. Find $\mathbf{a} + \mathbf{b}$ [2]



Exercise 2.1

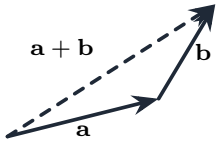
2.1 Write $\mathbf{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$. Find $\mathbf{a} + \mathbf{b}$ [2]



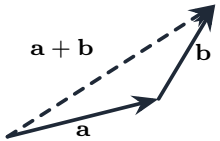
2.1 $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$.

Setup for exercise 2.2

2.2 Find $2\mathbf{a} - \mathbf{b}$ for the same vectors. [2]

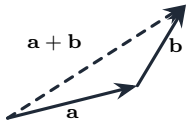


2.3 Find $|\mathbf{a}|$ for $\mathbf{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$. [2]

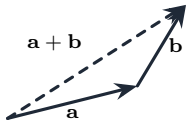


Exercise 2.2

2.4 $\overrightarrow{OA} = 3\mathbf{a}$, $\overrightarrow{AB} = 2\mathbf{b}$, $\overrightarrow{BC} = 2\mathbf{c}$. Write down $\overrightarrow{OC}$ [2]

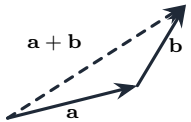


2.5 Write down $\overrightarrow{CO}$ in terms of $\overrightarrow{OC} = 3\mathbf{a} + 2\mathbf{b} + 2\mathbf{c}$. [1]

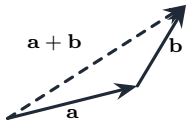


Exercise 2.2

2.4 $\overrightarrow{OA} = 3\mathbf{a}$, $\overrightarrow{AB} = 2\mathbf{b}$, $\overrightarrow{BC} = 2\mathbf{c}$. Write down $\overrightarrow{OC}$ [2]



2.5 Write down $\overrightarrow{CO}$ in terms of $\overrightarrow{OC} = 3\mathbf{a} + 2\mathbf{b} + 2\mathbf{c}$. [1]



2.2 $2\begin{pmatrix}4\\-3\end{pmatrix} - \begin{pmatrix}-1\\6\end{pmatrix} = \begin{pmatrix}8\\-6\end{pmatrix} - \begin{pmatrix}-1\\6\end{pmatrix} = \begin{pmatrix}9\\-12\end{pmatrix}$.

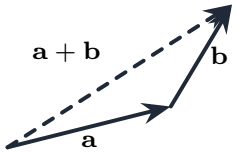
2.3 $\sqrt{16+9} = 5$.

2.4 $\overrightarrow{OC} = 3\mathbf{a} + 2\mathbf{b} + 2\mathbf{c}$ (route rule).

2.5 $\overrightarrow{CO} = -\overrightarrow{OC} = -3\mathbf{a} - 2\mathbf{b} - 2\mathbf{c}$.

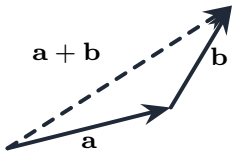
Exercise 3.1

3.1 *(Original.)* P, Q, R are such that $\overrightarrow{PQ} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and $\overrightarrow{QR} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$. (a) Find $\overrightarrow{PR}$. (b) Find $|\overrightarrow{PR}|$ to 3 significant figures. [4]



Exercise 3.1

3.1 *(Original.)* P, Q, R are such that $\overrightarrow{PQ} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and $\overrightarrow{QR} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$. (a) Find $\overrightarrow{PR}$. (b) Find $|\overrightarrow{PR}|$ to 3 significant figures. [4]

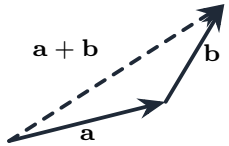


3.1 (a) $\overrightarrow{PR} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ 7 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$. (b) $\sqrt{16 + 25} = \sqrt{41} = 6.40$ (3 s.f.).

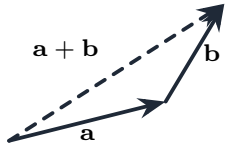
Exercise 3.2

3.2 *(Original.)* In triangle ABC, M is the midpoint of AB. $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$.

(a) Find $\overrightarrow{AM}$. (b) Find $\overrightarrow{AC}$. (c) Show that $\overrightarrow{MC} = \overrightarrow{AC} - \overrightarrow{AM}$ and hence find $\overrightarrow{MC}$. [5]



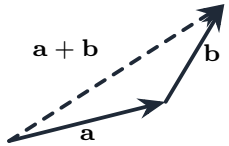
3.3 *(Original.)* $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$. The vector $\mathbf{u} = 2\mathbf{a} + k\mathbf{b}$ has modulus $5\sqrt{2}$. Find both possible values of k. [5]



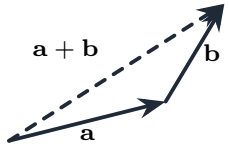
Exercise 3.2

3.2 *(Original.)* In triangle ABC, M is the midpoint of AB. $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$.

(a) Find $\overrightarrow{AM}$. (b) Find $\overrightarrow{AC}$. (c) Show that $\overrightarrow{MC} = \overrightarrow{AC} - \overrightarrow{AM}$ and hence find $\overrightarrow{MC}$. [5]



3.3 *(Original.)* $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$. The vector $\mathbf{u} = 2\mathbf{a} + k\mathbf{b}$ has modulus $5\sqrt{2}$. Find both possible values of k. [5]



Full solution on next slide

exercises.lol exercises.com.cn

Full solution 3.2

3.2 (a) $\overrightarrow{AM} = \frac{1}{2}\begin{pmatrix}6\\-4\end{pmatrix} = \begin{pmatrix}3\\-2\end{pmatrix}$. (b) $\overrightarrow{AC} = \begin{pmatrix}6\\-4\end{pmatrix} + \begin{pmatrix}-2\\8\end{pmatrix} = \begin{pmatrix}4\\4\end{pmatrix}$. (c) The route $A \to M \to C$ gives $\overrightarrow{MC} = \overrightarrow{AC} - \overrightarrow{AM} = \begin{pmatrix}4\\4\end{pmatrix} - \begin{pmatrix}3\\-2\end{pmatrix} = \begin{pmatrix}1\\6\end{pmatrix}$.

3.3 $\mathbf{u} = \begin{pmatrix}2\\4\end{pmatrix} + \begin{pmatrix}3k\\k\end{pmatrix} = \begin{pmatrix}2+3k\\4+k\end{pmatrix}$. The modulus condition gives $(2+3k)^2 + (4+k)^2 = 50$, so $4 + 12k + 9k^2 + 16 + 8k + k^2 = 50$, which simplifies to $10k^2 + 20k - 30 = 0$, then $k^2 + 2k - 3 = 0$. Factorising, $(k+3)(k-1) = 0$, so $k = 1$ or $k = -3$. Check $k = 1$: $\begin{pmatrix}5\\5\end{pmatrix}$ has modulus $\sqrt{50} = 5\sqrt{2}$. ✓

Local review guidance: the route written before components; exact column answers; modulus statements show the Pythagoras step. This is authored classroom guidance.