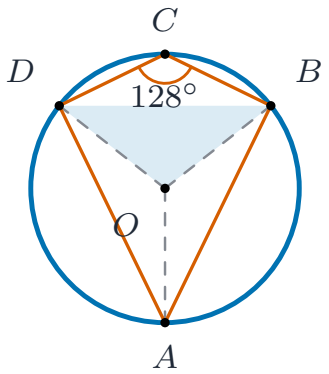
Exercise walk-through

4.5
Pythagoras and trigonometry in 2D

Pearson Edexcel · International GCSE · Mathematics A

Figure: Circle geometry with reasons (2)



cyclic quad: $\angle BAD = 52°$
centre $= 2\times$ rim: $\angle BOD = 104°$
$OB = OD$ (radii): isosceles
$\angle OBD = \frac{180-104}{2} = 38°$

every numerical step names its reason

- Worked example 2: a circle centre O with **cyclic quadrilateral** ABCD, angle BCD = 128°, and the chain BAD = 52°, BOD = 104°, OBD = 38°.
- Match each part of the diagram to the text.

Pythagoras and trigonometry in 2D

- $\tan 62° = \frac{h}{50}$, so $h = 50 \tan 62° = 94.0$ m (3 s.f.).
- The **sine rule** , $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, works when you know a side and its opposite angle.
- The **cosine rule** , $a^2 = b^2 + c^2 - 2bc\cos A$, works when you know two sides and the angle between them, or all three sides.

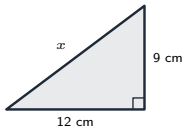
$$\sin\theta = \frac{\text{opp}}{\text{hyp}}, \quad \cos\theta = \frac{\text{adj}}{\text{hyp}}, \quad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

Pythagoras and trigonometry in 2D (2)

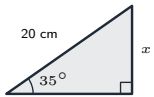
- The area of a triangle is $\frac{1}{2}ab\sin C$. It also works backwards: if the area is 15, $a = 6$ and $b = 10$, then $30\sin C = 15$, so $\sin C = 0.5$ and $C = 30°$ (or $150°$ if the angle is obtuse).
- $BC = a$ is opposite A, and $AC = b$ is opposite B. By the **sine rule**, $$\frac{b}{\sin 65°} = \frac{8}{\sin 40°}.$$
- $b = \dfrac{8\sin 65°}{\sin 40°} = 11.3$ cm (3 s.f.).

Exercises 2.1·2.2

2.1 Find the length x. [2]

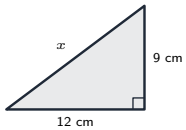


2.2 Find the length x. [2]



Exercises 2.1·2.2

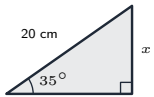
2.1 Find the length x. [2]



2.1

- $x^2 = 9^2 + 12^2 = 225$
- $x = \mathbf{15}$ cm

2.2 Find the length x. [2]

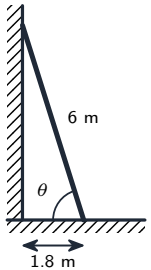


2.2

- $x = 20 \sin 35°$
- $x = \mathbf{11.5}$ cm

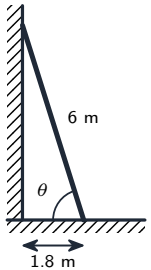
Exercise 2.3

2.3 A ladder 6 m long leans against a vertical wall. Its foot is 1.8 m from the wall on flat ground. Find the angle θ between the ladder and the ground. [2]



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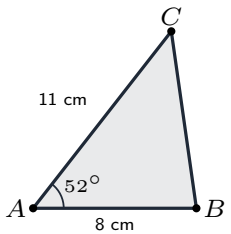


2.3

- $\cos\theta = \frac{1.8}{6}$
- $\theta = \mathbf{72.5}^\circ$

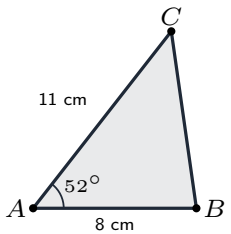
Exercise 3.1

3.1 In triangle ABC, $AB = 8$ cm, $AC = 11$ cm and angle $BAC = 52°$. Find the length BC. [3]



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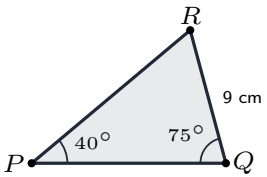


3.1

- $BC^2 = 8^2 + 11^2 - 2 \times 8 \times 11 \cos 52°$
- $BC^2 = 76.64$
- $BC = \mathbf{8.75}$ cm

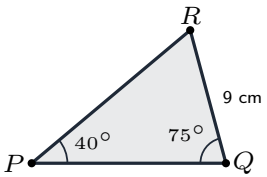
Exercise 3.2

3.2 In triangle PQR, angle $P = 40°$, angle $Q = 75°$ and $QR = 9$ cm. Find the length PR. [3]



Exercise 3.2

3.2 In triangle PQR, angle $P = 40°$, angle $Q = 75°$ and $QR = 9$ cm. Find the length PR. [3]



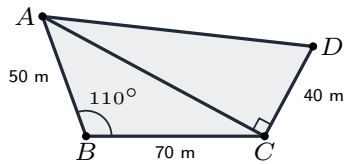
3.2

- $\dfrac{PR}{\sin 75°} = \dfrac{9}{\sin 40°}$
- $PR = \dfrac{9 \sin 75°}{\sin 40°}$
- $PR = \mathbf{13.5}$ cm

Exercise 3.3 (a)

3.3 The diagram shows a field $ABCD$. $AB = 50$ m, $BC = 70$ m and angle $ABC = 110°$. Angle $ACD = 90°$ and $CD = 40$ m.

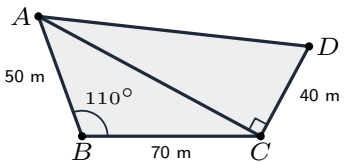
(a) Find the length AC. [3]



Exercise 3.3 (a)

3.3 The diagram shows a field $ABCD$. $AB = 50$ m, $BC = 70$ m and angle $ABC = 110°$. Angle $ACD = 90°$ and $CD = 40$ m.

(a) Find the length AC. [3]



(a) $AC^2 = 50^2 + 70^2 - 2 \times 50 \times 70 \cos 110° = 9794$; $AC =$ **99.0** m

Exercise 3.3 (b)

3.3 The diagram shows a field $ABCD$. $AB = 50$ m, $BC = 70$ m and angle $ABC = 110°$. Angle $ACD = 90°$ and $CD = 40$ m.

(b) Find the size of angle BAC. [3]

Exercise 3.3 (b)

3.3 The diagram shows a field $ABCD$. $AB = 50$ m, $BC = 70$ m and angle $ABC = 110°$. Angle $ACD = 90°$ and $CD = 40$ m.

(b) Find the size of angle BAC. [3]

(b) $\dfrac{\sin BAC}{70} = \dfrac{\sin 110°}{98.97}$; $\sin BAC = 0.6647$; angle $BAC = \mathbf{41.7}°$

Exercise 3.3 (c)

3.3 The diagram shows a field $ABCD$. $AB = 50$ m, $BC = 70$ m and angle $ABC = 110°$. Angle $ACD = 90°$ and $CD = 40$ m.

(c) Find the length AD and the size of angle CAD. [4]

Exercise 3.3 (c)

3.3 The diagram shows a field $ABCD$. $AB = 50$ m, $BC = 70$ m and angle $ABC = 110°$. Angle $ACD = 90°$ and $CD = 40$ m.

(c) Find the length AD and the size of angle CAD. [4]

(c) $AD^2 = 98.97^2 + 40^2$; $AD = \mathbf{107}$ m; $\tan CAD = \frac{40}{98.97}$; angle $CAD = \mathbf{22.0}°$

Exercise 3.3 (d)

3.3 The diagram shows a field $ABCD$. $AB = 50$ m, $BC = 70$ m and angle $ABC = 110°$. Angle $ACD = 90°$ and $CD = 40$ m.

(d) Find the area of the whole field. Grass seed is sold in bags. One bag covers 250 m^2. How many bags are needed for the whole field? [5]

Exercise 3.3 (d)

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(d) Find the area of the whole field. Grass seed is sold in bags. One bag covers 250 m^2. How many bags are needed for the whole field? [5]

(d) triangle ABC: $\frac{1}{2} \times 50 \times 70 \sin 110° = 1644$ m^2; triangle ACD: $\frac{1}{2} \times 98.97 \times 40 = 1979$ m^2; total **3620** m^2; $3624 \div 250 = 14.5$, so **15** bags