Exercise walk-through

2.4
Completing the square and solving quadratics

Pearson Edexcel · International GCSE · Mathematics A

Quadratic algebra

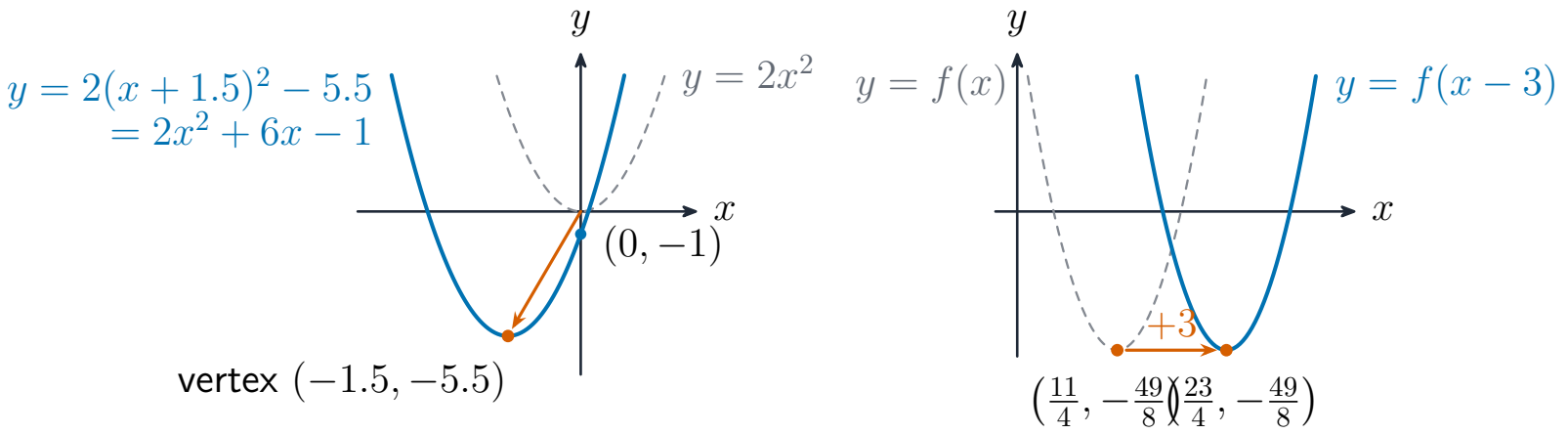
- To factorise $x^2 + bx + c$, find two numbers that multiply to c and add to b: $x^2 + 10x + 24 = (x+6)(x+4)$.
- (Higher) To factorise $ax^2 + bx + c$, find two numbers that multiply to ac and add to b, then split the middle term: $6x^2 - 5x - 6 = 6x^2 - 9x + 4x - 6 = 3x(2x-3) + 2(2x-3) = (3x+2)(2x-3)$.
- The **difference of two squares** : $x^2 - a^2 = (x-a)(x+a)$, so $4x^2 - 9 = (2x-3)(2x+3)$.

$$\frac{x^2-4}{x^2+5x+6} = \frac{(x-2)(x+2)}{(x+2)(x+3)} = \frac{x-2}{x+3}$$

Completing the square and solving quadratics (2)

- factorise: $x^2 + x - 30 = 0$ gives $(x+6)(x-5) = 0$, so the **roots** are $x = -6$ and $x = 5$;
- use the **quadratic formula** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ when it does not factorise;
- complete the square.
- $a = 2$, $b = 6$, $c = -1$: $x = \dfrac{-6 \pm \sqrt{36+8}}{4} = \dfrac{-6 \pm 2\sqrt{11}}{4} = \dfrac{-3 \pm \sqrt{11}}{2}$.

Figure: Completing the square and solving quadratics

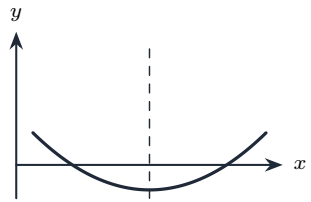


$a(x + p)^2 + q$ has its vertex at $(-p, q)$; $f(x - 3)$ moves every point 3 to the right

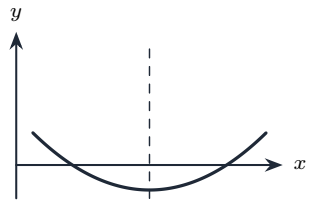
- Left: **completing the square** shows the **vertex** of $y = 2x^2 + 6x - 1$ at $x = -1.5$. Right: worked example 9, the minimum moving 3 to the right under $f(x - 3)$.
- Match each part of the diagram to the text.

Exercises 2.1·2.2

2.1 Write $x^2 + 6x + 5$ in the form $(x + b)^2 + c$ [2]

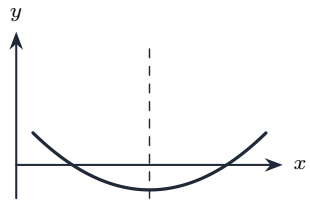


2.2 Hence state the coordinates of the minimum point of $y = x^2 + 6x + 5$ [1]



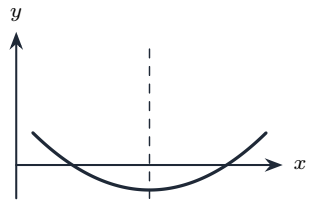
Exercises 2.1·2.2

2.1 Write $x^2 + 6x + 5$ in the form $(x + b)^2 + c$ [2]



2.1 $x^2 + 6x + 5 = (x + 3)^2 - 9 + 5 = (x + 3)^2 - 4$.

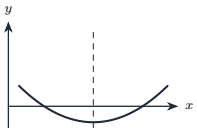
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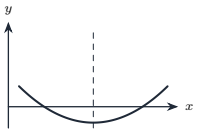
2.2 Minimum $(-3, -4)$.

Exercise 2.3

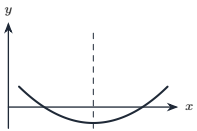
2.3 Solve $x^2 + x - 30 = 0$ by factorisation [2]



2.4 Solve $2x^2 + 6x - 1 = 0$, giving exact answers [3]

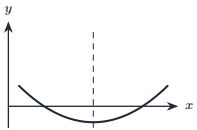


2.5 Write $2x^2 - 11x + 9$ in the form $a(x + b)^2 + c$ [3]

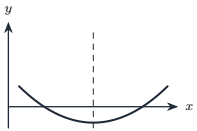


Exercise 2.3

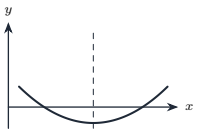
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2.3 $(x + 6)(x - 5) = 0$: $x = -6$ or 5.

2.4 $x = \dfrac{-6 \pm \sqrt{36 + 8}}{4} = \dfrac{-6 \pm \sqrt{44}}{4} = \dfrac{-3 \pm \sqrt{11}}{2}$.

Full solution on next slide

Full solution 2.3

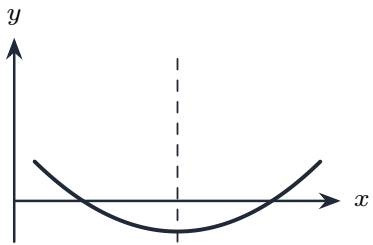
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2.4 $x=\dfrac{-6\pm\sqrt{36+8}}{4}=\dfrac{-6\pm\sqrt{44}}{4}=\dfrac{-3\pm\sqrt{11}}{2}$.

2.5 $2x^2-11x+9=2\left(x-\frac{11}{4}\right)^2-\frac{121}{8}+9=2\left(x-\frac{11}{4}\right)^2-\frac{49}{8}$.

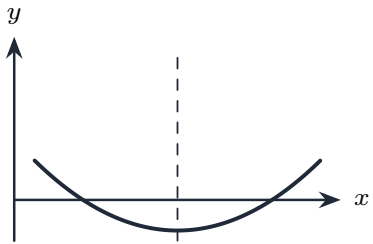
Exercise 3.1

3.1 *(Edexcel 4MA1 1HR June 2024 Q20, adapted.)* (a) Write $2x^2 - 11x + 9$ in the form $a(x+b)^2 + c$. (b) The graph of $y = f(x-3)$, where $f(x) = 2x^2 - 11x + 9$, has a minimum point. Find the coordinates of this minimum point. [5]



Exercise 3.1

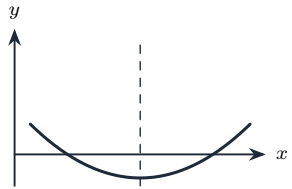
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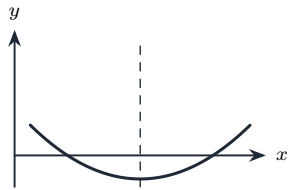
3.1 (a) As 2.5: $2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8}$. (b) f has its minimum at $\left(\frac{11}{4}, -\frac{49}{8}\right)$; $f(x-3)$ shifts 3 right, so the minimum is $\left(\frac{23}{4}, -\frac{49}{8}\right) = (5.75, -6.125)$.

Exercise 3.2

3.2 *(Original.)* A rectangular lawn has length 3 m more than its width. Its area is 70 m^2. Form and solve a quadratic equation to find the width and length. [4]

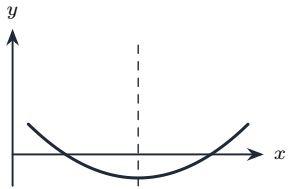


3.3 *(Original.)* Solve $3x^2 = 4x + 2$ giving answers correct to 2 decimal places. [3]

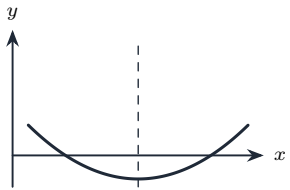


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3.2 Width w: $w(w+3) = 70$, so $w^2 + 3w - 70 = 0 = (w+10)(w-7)$. Width 7 m (positive root), length 10 m. Check: $7 \times 10 = 70$. ✓

Full solution on next slide

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Full solution 3.2

3.2 Width w: $w(w+3) = 70$, so $w^2 + 3w - 70 = 0 = (w+10)(w-7)$. Width 7 m (positive root), length 10 m. Check: $7 \times 10 = 70$. ✓

3.3 $3x^2 - 4x - 2 = 0$: $x = \dfrac{4 \pm \sqrt{16+24}}{6} = \dfrac{4 \pm \sqrt{40}}{6} = 1.72$ or -0.39 (2 d.p.).

Local review guidance: half-the-coefficient structure with the correcting constant; exact values unless decimals are asked; context rejects the impossible root with a reason. This is authored classroom guidance.