

Course notes

Pearson Edexcel · International GCSE · Mathematics A

4MA1: course teaching notes

Version: Issue 2, November 2017; first assessment June 2018; linear Mathematics A

These are original teaching notes, and each lesson is labelled with its tier.

1 · Exact arithmetic and estimation · Foundation

Can we pack without leftovers?

- A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers?
- This lesson studies **prime factor** : A prime number that divides the integer exactly.

The method

- Prime factors reveal shared structure.
- Use the smallest common prime powers for the HCF and the largest for the LCM.
- Estimate before calculating; use brackets to preserve the order of operations.

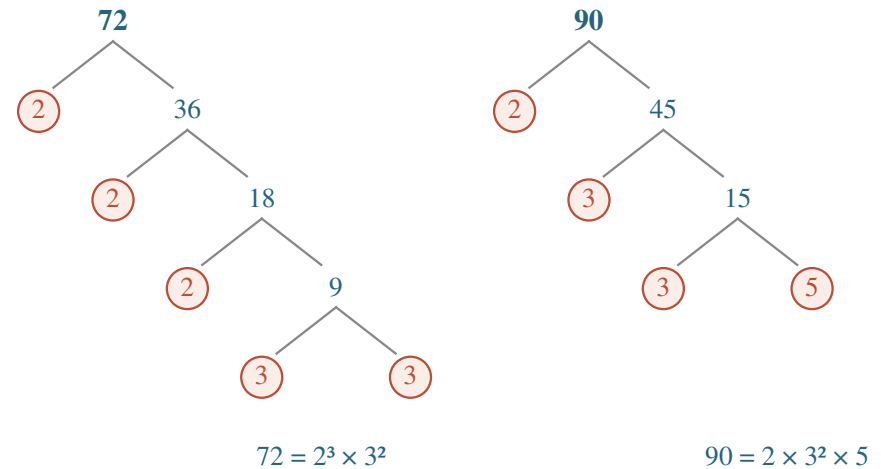
A worked case

$$72 = 2^3 \times 3^2, \quad 90 = 2 \times 3^2 \times 5, \quad \text{HCF} = 18$$

■ Worked example

$72 = 2^3 \times 3^2$ and $90 = 2 \times 3^2 \times 5$. Their HCF is $2 \times 9 = 18$. Make 18 bags with 4 pencils and 5 pens each. Their LCM is $2^3 \times 3^2 \times 5 = 360$.

Factor trees: $\text{HCF}(72, 90) = 2 \times 3^2 = 18$



Where it goes wrong

- The HCF divides both numbers; the LCM is a multiple of both.
- They answer different questions.
- A decimal estimate is not an exact fraction.

■ Watch out

Not true: The HCF of two positive integers is larger than either integer. The HCF divides both numbers, so it can never be larger than the smaller one: the HCF of 72 and 90 is 18.

Applying it

- For a non-calculator paper, keep fractions exact and show cancellation.
- For a calculator paper, enter the full expression and compare with your estimate.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Foundation tier, specification section 1.

■ Key idea

- **Prime factor:** A prime number that divides the integer exactly.
- Prime factors reveal shared structure.
- Watch for: the HCF divides both numbers; the LCM is a multiple of both.

1 · Exact arithmetic and estimation · Higher

Can we pack without leftovers?

- A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers?
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The method

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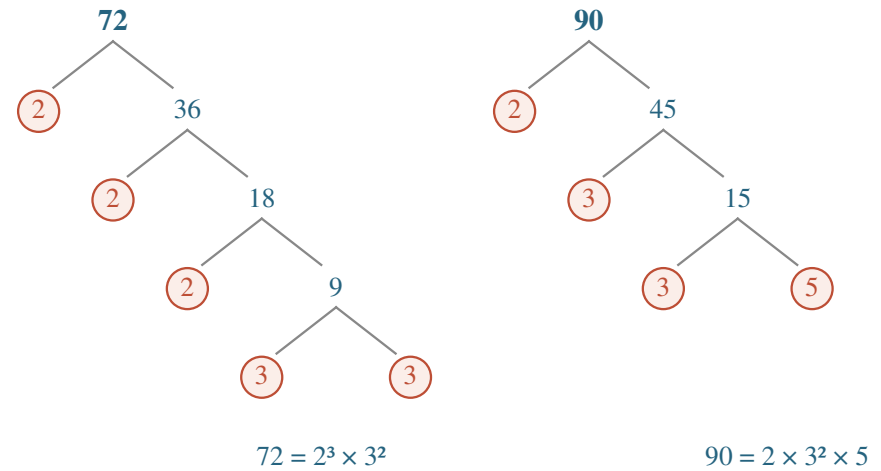
A worked case

$$a = \prod p_i^{\alpha_i}, \quad b = \prod p_i^{\beta_i}, \quad \text{HCF}(a, b) = \prod p_i^{\min(\alpha_i, \beta_i)}$$

■ Worked example

$72 = 2^3 \times 3^2$ and $90 = 2 \times 3^2 \times 5$. Their HCF is $2 \times 9 = 18$. Make 18 bags with 4 pencils and 5 pens each. Their LCM is $2^3 \times 3^2 \times 5 = 360$.

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In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 1.

■ Key idea

- **Prime factor:** A prime number that divides the integer exactly.
- Prime factors reveal shared structure.
- Watch for: the HCF divides both numbers; the LCM is a multiple of both.

1 · Integer indices and standard form · Foundation

How small is a microscopic length?

- A microscope records a length of 0.000072 metres. A compact representation must keep its size correct.
- This lesson studies **index** : The power to which a base is raised.

The method

- Use integer powers, square and cube roots and standard form with $1 \leq a < 10$.
- In multiplying powers with the same base, add indices; in division, subtract them.

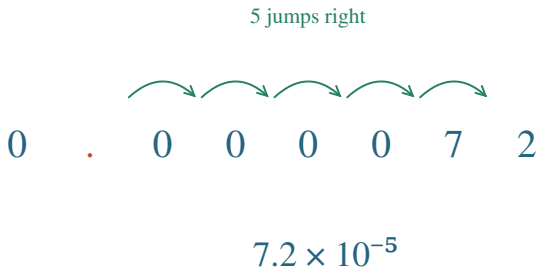
A worked case

$a^m a^n = a^{m+n}$

■ Worked example

0.000072=7.2×10⁽⁻⁵⁾. Also 2³×2⁴=2⁷=128. The square root of 81 is 9. Check a standard-form answer by writing it out as a decimal.

0.000072 = 7.2 × 10⁻⁵: the point moves 5 places



coefficient 7.2 is between 1 and 10; the power is -5

Where it goes wrong

- Index laws do not turn a sum into a single power: 2³+2⁴=24, not 2⁷.
- Do not round a surd when an exact answer is requested.

■ Watch out

Not true: For every positive a, a²+a³ equals a⁵. Index laws only add powers when the terms are multiplied: 2²+2³=12, but 2⁵=32.

Applying it

- This Foundation/Core lesson excludes fractional powers and surd rationalisation.
- Estimate a result before using a calculator and retain the required precision.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Foundation tier, specification section 1.

■ Key idea

- **Index:** The power to which a base is raised.
- Use integer powers, square and cube roots and standard form with $1 \leq a < 10$.
- Watch for: index laws do not turn a sum into a single power: 2³+2⁴=24, not 2⁷.

1 · Indices, surds and standard form · Higher

How small is a microscopic length?

- A microscope records a length of 0.000072 metres. A compact representation must keep its size correct.
- This lesson studies **index** : The power to which a base is raised.

The method

- For the same positive base, multiplication adds indices and division subtracts them.
- A negative index means reciprocal; a fractional index represents a root.
- Standard form has $1 \leq a < 10$.

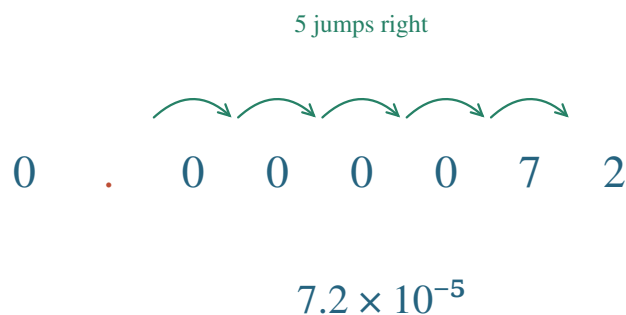
A worked case

$$a^m a^n = a^{m+n}, \quad a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = (\sqrt[n]{a})^m$$

■ Worked example

$0.000072 = 7.2 \times 10^{-5}$. Also $16^{3/4} = (16^{1/4})^3 = 2^3 = 8$. Simplify $\sqrt{72} = 6\sqrt{2}$, then rationalise $1/\sqrt{2} = \sqrt{2}/2$.

$0.000072 = 7.2 \times 10^{-5}$: the point moves 5 places



coefficient 7.2 is between 1 and 10; the power is -5

Where it goes wrong

- Index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 .
- Do not round a surd when an exact answer is requested.

■ Watch out

Not true: For every positive a , $a^2 + a^3$ equals a^5 . Index laws only add powers when the terms are multiplied: $2^2 + 2^3 = 12$, but $2^5 = 32$.

Applying it

- Check powers of ten against the original quantity.
 - Use surds for exact geometry, and round only the final length when the question asks for a decimal.
-

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Index:** The power to which a base is raised.
- For the same positive base, multiplication adds indices and division subtracts them.
- Watch for: index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 .

1 · Percentages, ratio and proportional reasoning · Foundation

Can we recover the original price?

- A coat is reduced by 20% to ¥240. The discount applies to the original price, not to the sale price.
 - This lesson studies **multiplier** : A factor that performs a percentage change in one multiplication.
-

The method

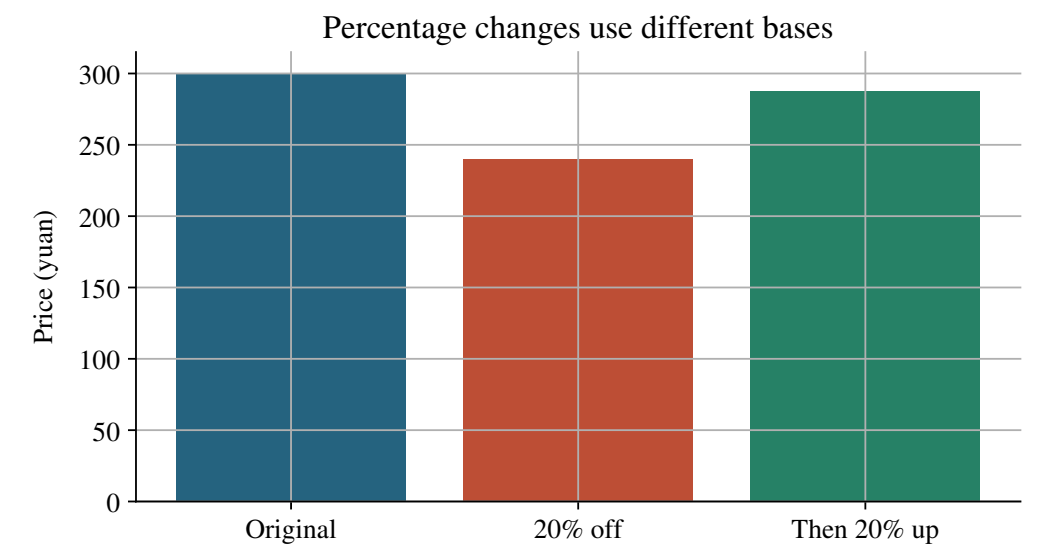
- A $p\%$ increase has multiplier $1 + p/100$; a decrease has multiplier $1 - p/100$.
 - Reverse a percentage by dividing by the multiplier.
 - In a ratio, first find the total number of parts.
-

A worked case

$$P_{\text{new}} = P_{\text{old}} \left(1 + \frac{r}{100}\right)$$

■ **Worked example**

Let the original price be P. The model is sale price=0.8P. Hence P=240/0.8=300. A later 20% increase gives 240×1.2=288, so the two changes do not cancel.



Where it goes wrong

- A percentage uses a stated base.
 - Subtracting the percentages loses that base.
 - For compound change, multiply the multipliers; do not add the percentages.
- **Watch out**
- Not true:** A 20% decrease followed by a 20% increase restores the starting price. The 20% increase is taken of a smaller amount: $100 \times 0.8 \times 1.2 = 96$, not 100.

Applying it

- Use percentage multipliers and divide a total into ratio parts.
- This Foundation/Core lesson uses linear proportional contexts, not the advanced regression methods.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Foundation tier, specification section 1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Multiplier:** A factor that performs a percentage change in one multiplication.
- A p% increase has multiplier $1 + p/100$; a decrease has multiplier $1 - p/100$.
- Watch for: A percentage uses a stated base.

1 · Percentages, ratio and proportional reasoning · Higher

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The method

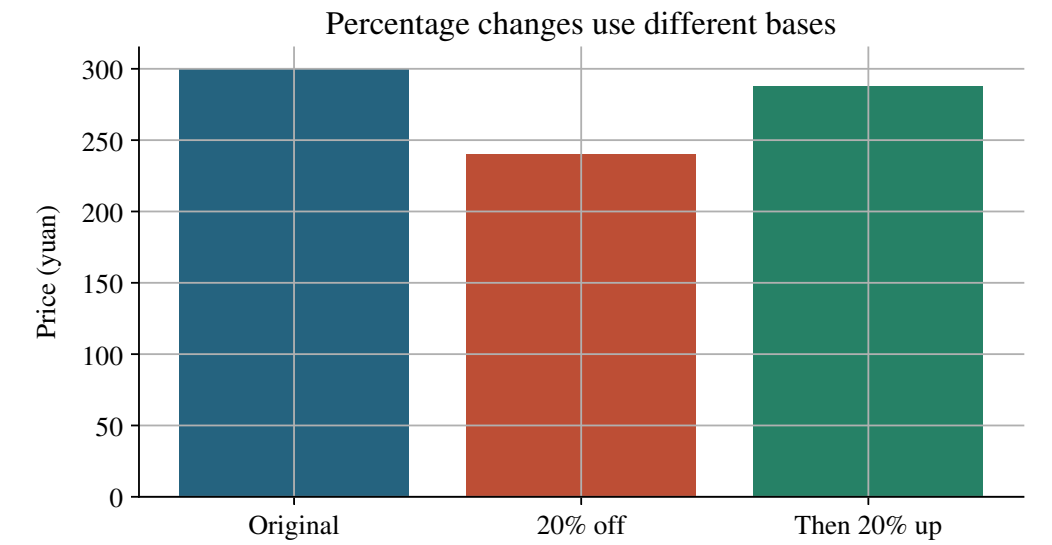
- A p% increase has multiplier $1 + p/100$; a decrease has multiplier $1 - p/100$.
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A worked case

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■ **Watch out**

Not true: A 20% decrease followed by a 20% increase restores the starting price. The 20% increase is taken of a smaller amount: $100 \times 0.8 \times 1.2 = 96$, not 100.

Applying it

- For direct proportion use $y=kx$; for inverse proportion use $y=k/x$.
- Calculate k from a known pair before using a new value.
- State what you held constant.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ **Key idea**

- **Multiplier:** A factor that performs a percentage change in one multiplication.
- A p% increase has multiplier $1+p/100$; a decrease has multiplier $1-p/100$.
- Watch for: A percentage uses a stated base.

1 · Accuracy, bounds and compound measures · Higher

Is the printed measurement exact?

- A rectangular panel is labelled 8.0 cm by 5.0 cm, each to the nearest 0.1 cm. Its true area is not fixed at 40 cm².
- This lesson studies **lower bound** : The smallest possible value consistent with a stated rounding rule.

The method

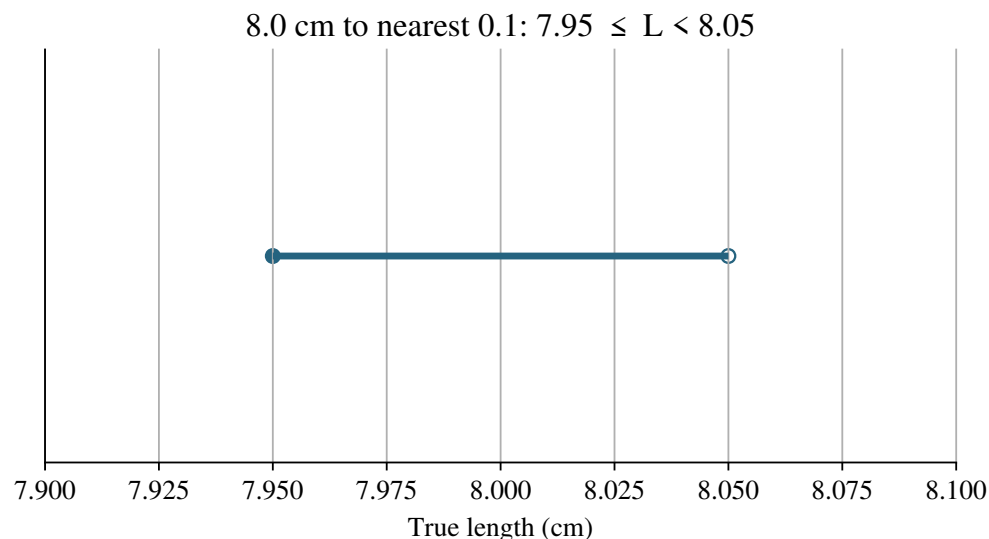
- A value rounded to the nearest unit u lies from stated value-u/2 up to, but usually not including, stated value+u/2.
- For positive quantities, combine extremes according to the operation.

A worked case

$$A = LW, \quad v = \frac{d}{t}$$

■ **Worked example**

The lengths satisfy $7.95 \leq L < 8.05$ and $4.95 \leq W < 5.05$. Since $A=LW$, $39.3525 \leq A < 40.6525$. For speed d/t , the largest speed uses the largest distance and smallest positive time.



Where it goes wrong

- An upper bound is not automatically achieved.
- Dividing upper distance by upper time does not give the largest speed.
- Keep enough digits in intermediate calculations.

■ Watch out

Not true: The greatest value of positive d/t uses the greatest d and greatest t . To make d/t as large as possible, use the greatest d and the least t ; a larger t makes the quotient smaller.

Applying it

- Distinguish measurement uncertainty from arithmetic rounding.
- A sensible reported precision cannot be finer than the measurements justify.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Lower bound:** The smallest possible value consistent with a stated rounding rule.
- A value rounded to the nearest unit u lies from stated value- $u/2$ up to, but usually not including, stated value+ $u/2$.
- Watch for: an upper bound is not automatically achieved.

2 · Equations, identities and rearrangement · Foundation

When do two plans cost the same?

- Two mobile plans cost $20+3x$ and $44+x$ yuan for x GB. When do they cost the same?
- This lesson studies **identity** : An equality that holds for every allowed value of its variable.

The method

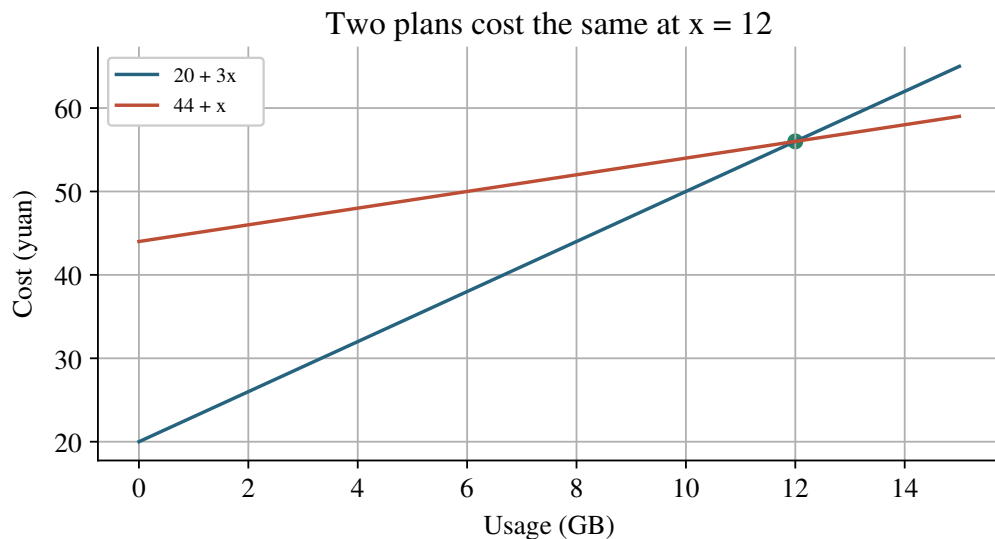
- An equation asks which inputs satisfy an equality; an identity holds for all allowed inputs.
- Preserve equality by applying the same operation to both sides.
- State restrictions before dividing by a variable.

A worked case

$$C_1 = 20 + 3x, \quad C_2 = 44 + x, \quad C_1 = C_2$$

■ Worked example

$20+3x=44+x$ gives $2x=24$ and $x=12$. Both plans then cost 56. In $A=\pi r^2$, divide by π and take the positive square root to obtain $r=\sqrt{A/\pi}$, because r is a length.



Where it goes wrong

- Cancelling a term is not the same as cancelling a factor.
- In $(x^2+2x)/x$, factor the numerator and retain $x \neq 0$.
- Check a rearrangement by substitution.

■ Watch out

Not true: Cancelling x from $(x+3)/x$ leaves 3 for every nonzero x . x is a term of the numerator, not a factor, so it cannot be cancelled: at $x=1$, $(1+3)/1=4$, not 3.

Applying it

- Define the unknown and set up a linear equation.
- Check the answer by substitution.
- Restrict this Foundation/Core lesson to simple expressions and equations.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Foundation tier, specification section 2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Identity:** An equality that holds for every allowed value of its variable.
- An equation asks which inputs satisfy an equality; an identity holds for all allowed inputs.
- Watch for: cancelling a term is not the same as cancelling a factor.

2 · Equations, identities and rearrangement · Higher

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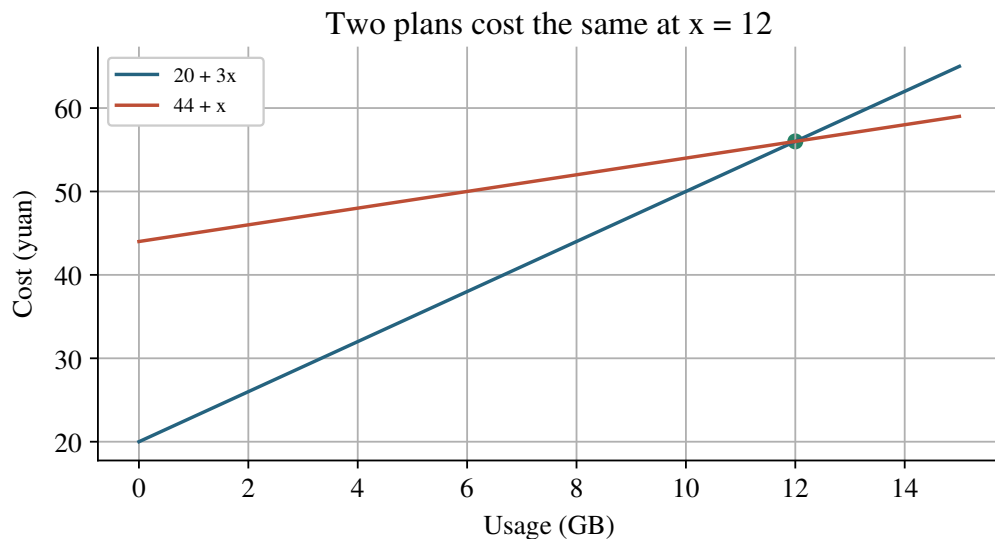
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Not true: Cancelling x from $(x+3)/x$ leaves 3 for every nonzero x . x is a term of the numerator, not a factor, so it cannot be cancelled: at $x=1$, $(1+3)/1=4$, not 3.

Applying it

- Set up the equation from units and the meaning of the unknown.
- A negative or fractional solution may be algebraically correct but impossible for a count.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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- An equation asks which inputs satisfy an equality; an identity holds for all allowed inputs.
- Watch for: cancelling a term is not the same as cancelling a factor.

2 · Factorising and solving simple quadratics · Foundation

Which widths make enough space?

- A rectangular enclosure has area $x(10-x)$. What widths give at least 21 square metres?
- This lesson studies **discriminant**: The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$.

The method

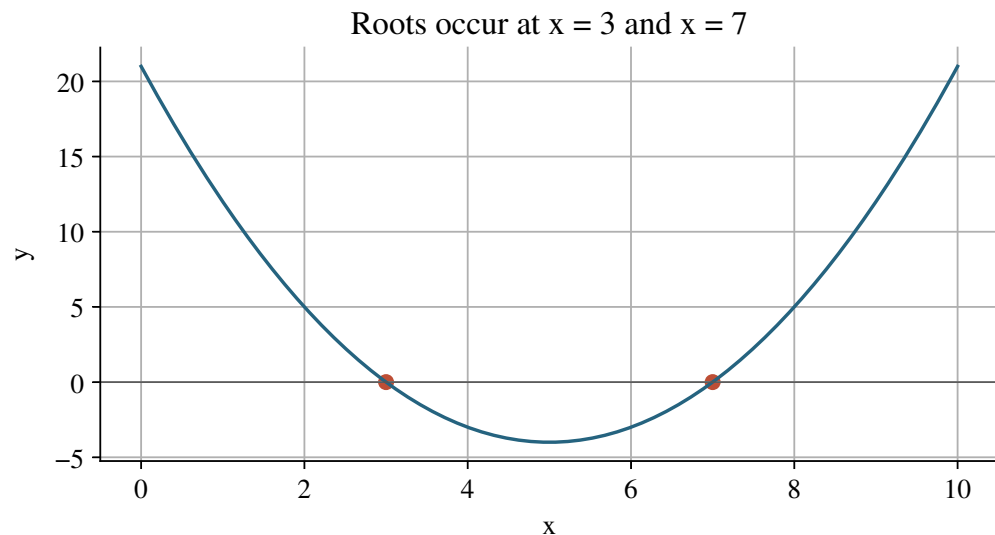
- Expand brackets and factorise simple quadratics.
- Solve by setting each factor equal to zero, and use a graph to interpret the roots.

A worked case

$$(x-3)(x-7)=0$$

■ Worked example

$x^2-10x+21=(x-3)(x-7)$. Hence the equation $x^2-10x+21=0$ has roots 3 and 7. Check each root by substitution and mark both intercepts on the graph.



Where it goes wrong

- Multiplying an inequality by a negative number reverses its direction.
- A sketch must show which side of each root satisfies the inequality.
- Geometry may restrict x further.

■ Watch out

Not true: A positive discriminant means that a quadratic has no real roots. A positive discriminant means two distinct real roots; a negative discriminant means no real roots.

Applying it

- This Foundation/Core lesson uses factorisation and graphical roots; the discriminant, quadratic formula and quadratic inequalities are reserved for the advanced tier.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Foundation tier, specification section 2.

- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Discriminant:** The quantity $b^2 - 4ac$ that determines the real roots of $ax^2 + bx + c = 0$.
- Expand brackets and factorise simple quadratics.
- Watch for: multiplying an inequality by a negative number reverses its direction.

2 · Quadratics and inequalities · Higher

Which widths make enough space?

- A rectangular enclosure has area $x(10-x)$. What widths give at least 21 square metres?
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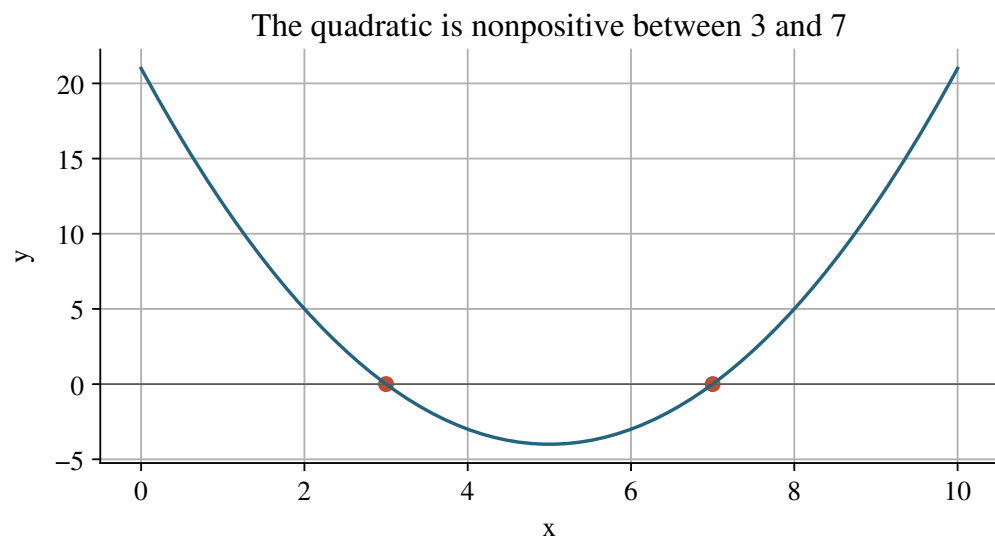
- Factor where possible; otherwise complete the square or use the quadratic formula.
- A quadratic inequality needs the sign on intervals, not only the roots.
- The discriminant identifies repeated or missing real roots.

A worked case

$$ax^2 + bx + c = 0, \quad \Delta = b^2 - 4ac$$

■ Worked example

The condition is $x(10-x) \geq 21$, so $x^2 - 10x + 21 \leq 0$. Factor $(x-3)(x-7) \leq 0$. The upward parabola is nonpositive between its roots, giving $3 \leq x \leq 7$. The maximum area is 25 at $x=5$.



Where it goes wrong

- Multiplying an inequality by a negative number reverses its direction.
- A sketch must show which side of each root satisfies the inequality.
- Geometry may restrict x further.

■ Watch out

Not true: A positive discriminant means that a quadratic has no real roots. A positive discriminant means two distinct real roots; a negative discriminant means no real roots.

Applying it

- Use the vertex to interpret an optimum.
- Check an endpoint and a point between the roots; the algebra and graph should tell the same story.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 2.

- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Discriminant:** The quantity $b^2 - 4ac$ that determines the real roots of $ax^2 + bx + c = 0$.
- Factor where possible; otherwise complete the square or use the quadratic formula.
- Watch for: multiplying an inequality by a negative number reverses its direction.

2 · Simultaneous equations and feasible regions · Foundation

Can one equation determine two counts?

- Two ticket types raise a total amount. A single equation cannot identify both unknown counts.
- This lesson studies **elimination** : Combining equations to remove one variable while retaining the same solutions.

The method

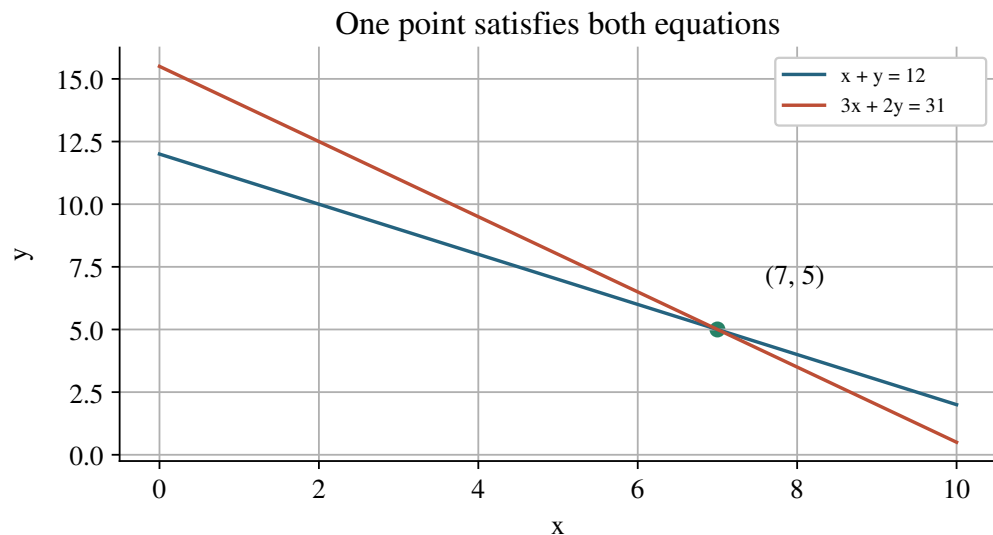
- For two linear equations, use elimination or substitution and check both equations.
- For a line and a quadratic, substitute the linear relation first; then solve the resulting quadratic.
- For inequalities, shade the region satisfying every condition.

A worked case

$$x + y = 12, \quad 3x + 2y = 31$$

■ Worked example

If $x + y = 12$ and $3x + 2y = 31$, subtract twice the first equation from the second to obtain $x = 7$, then $y = 5$. For $y = x + 1$ and $y = x^2 - 1$, $x^2 - x - 2 = 0$ gives $x = 2$ or -1 , with $y = 3$ or 0 .



Where it goes wrong

- One equation checked is not enough.
- A line can meet a quadratic twice, so retain both solutions unless the context removes one.
- Inequality boundaries may be included or excluded according to the sign.

■ Watch out

Not true: A pair of simultaneous equations is solved by checking just one of them. A solution must satisfy both equations; check it in each one.

Applying it

- Define the variables and their units.
- If they count objects, both must be nonnegative integers.
- For a feasible region, test one point on the required side of each boundary and then take the intersection.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Foundation tier, specification section 2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Elimination:** Combining equations to remove one variable while retaining the same solutions.
- For two linear equations, use elimination or substitution and check both equations.
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2 · Simultaneous equations and feasible regions · Higher

Can one equation determine two counts?

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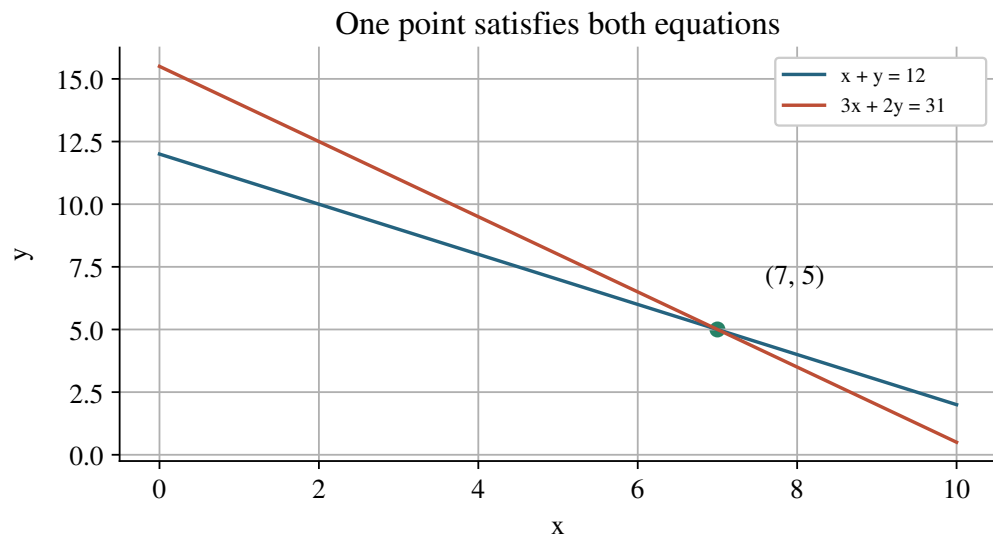
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Applying it

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In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Elimination:** Combining equations to remove one variable while retaining the same solutions.
- For two linear equations, use elimination or substitution and check both equations.
- Watch for: one equation checked is not enough.

3 · Arithmetic sequences and nth terms · Foundation

Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
- This lesson studies **common difference**: The constant added between consecutive terms of an arithmetic sequence.

The method

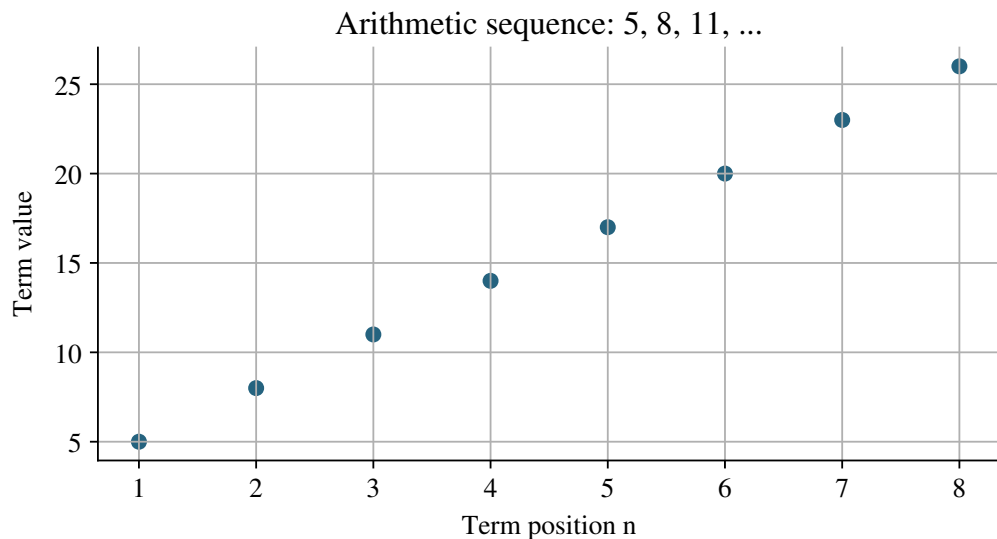
- Find a constant difference for an arithmetic sequence.
- Its n th term is $a + (n-1)d$.
- A term-to-term rule describes how to reach the next term; a position-to-term rule gives a term directly.

A worked case

$$u_n = a + (n - 1)d$$

■ Worked example

For 5, 8, 11, 14, ... the common difference is 3. The n th term is $5 + 3(n-1) = 3n + 2$. At $n=8$, $u_8=26$. To find the position of 62, solve $3n + 2 = 62$, giving $n=20$.



Where it goes wrong

- The first term has index 1, so the exponent is $n-1$.
- A sequence is a list; a series is a sum.
- A geometric sequence can alternate in sign and still converge.

■ Watch out

Not true: The first term of a sequence has index zero. A sum to infinity exists only when $-1 < r < 1$; for $r=2$ the terms grow without limit.

Applying it

- Generate several terms and check a proposed n th-term rule.
- Infinite geometric series and advanced sum formulae are excluded from this Foundation/Core lesson.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Foundation tier, specification section 3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Common difference:** The constant added between consecutive terms of an arithmetic sequence.
- Find a constant difference for an arithmetic sequence.
- Watch for: the first term has index 1, so the exponent is $n-1$.

3 · Sequences, series and recurrence · Higher

Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
- This lesson studies **common ratio**: The constant multiplier between consecutive terms of a geometric sequence.

The method

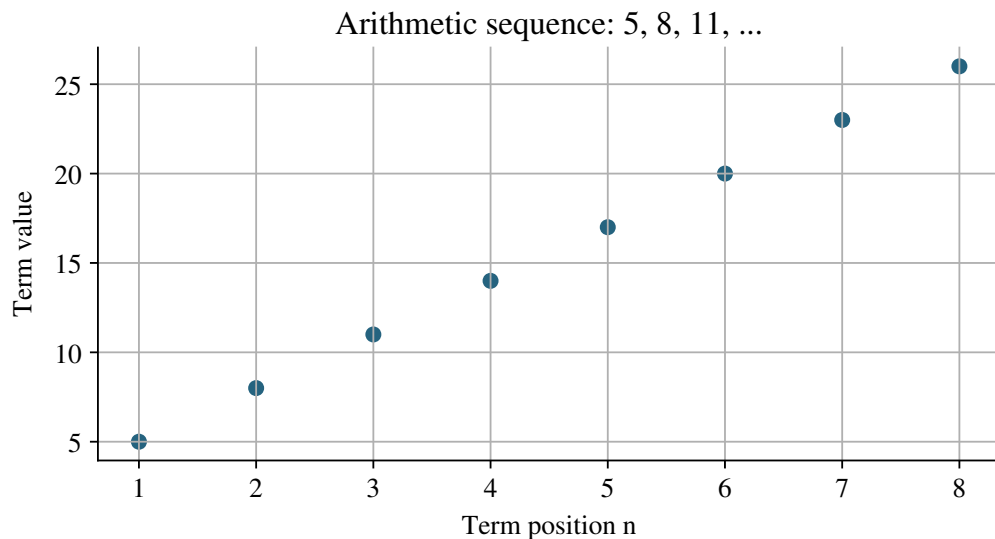
- For an arithmetic progression, $u_n = a + (n-1)d$ and $S_n = n(2a + (n-1)d)/2$.
- For a geometric progression, $u_n = ar^{n-1}$ and $S_n = a(1-r^n)/(1-r)$.
- An infinite geometric sum exists only if $|r| < 1$.

A worked case

$$u_n = a + (n-1)d, \quad S_n = \frac{n}{2} [2a + (n-1)d]$$

■ Worked example

For $a=5, d=3, n=8$, $u_8 = 5 + 7 \times 3 = 26$ and $S_8 = 8(10+21)/2 = 124$. For $a=12, r=1/2$, $S_{\infty} = 12/(1-1/2) = 24$. For $u_{n+1} = 2u_n + 1$ with $u_1 = 1$, the next terms are 3, 7, 15.



Where it goes wrong

- The first term has index 1, so the exponent is $n-1$.
- A sequence is a list; a series is a sum.
- A geometric sequence can alternate in sign and still converge.

■ Watch out

Not true: Every geometric series has a finite sum to infinity. A sum to infinity exists only when $-1 < r < 1$; for $r=2$ the terms grow without limit.

Applying it

- Explain whether the context justifies additive or multiplicative change.
- In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Common ratio:** The constant multiplier between consecutive terms of a geometric sequence.
- For an arithmetic progression, $u_n = a + (n-1)d$ and $S_n = n(2a + (n-1)d)/2$.
- Watch for: the first term has index 1, so the exponent is $n-1$.

3 · Straight lines and gradients · Foundation

How does a path's slope become an equation?

- A path rises 6 metres over a horizontal distance of 3 metres. Its gradient connects a diagram to an equation.
- This lesson studies **gradient**: The change in y divided by the corresponding change in x .

The method

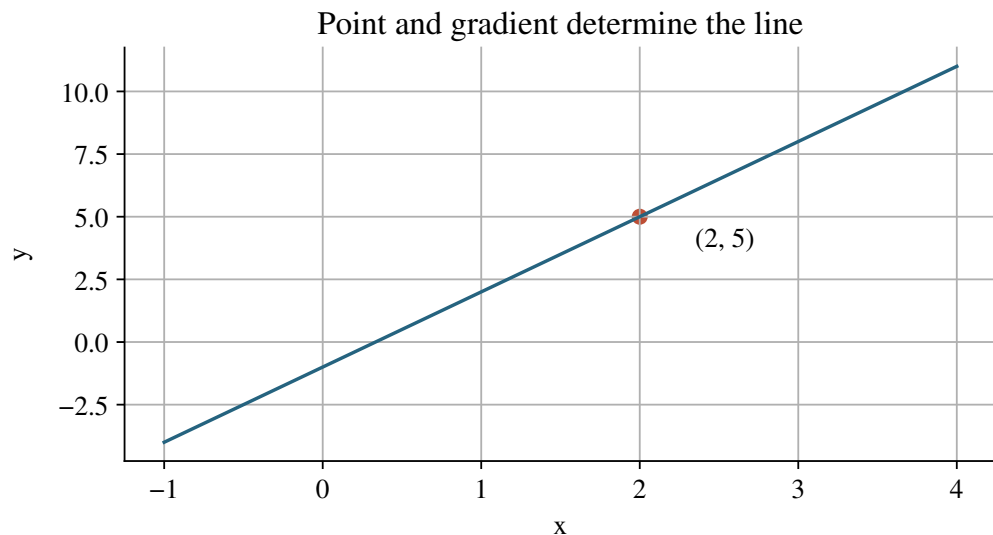
- Gradient is change in y divided by change in x .
- A straight line has $y = mx + c$, where c is its y -intercept.
- Parallel lines have equal gradients.

A worked case

$$y = mx + c$$

■ Worked example

Through (2,5) with gradient 3, substitute to get $5 = 3 \times 2 + c$, so $c = -1$ and $y = 3x - 1$. Points (1,2) and (4,8) give gradient $(8-2)/(4-1) = 2$.



Where it goes wrong

- A vertical line has no finite gradient; do not force it into $y=mx+c$.
- Read the signs of a circle's centre carefully.
- The radius to a tangent is perpendicular to the tangent.

■ Watch out

Not true: Perpendicular nonvertical lines have equal gradients. Perpendicular gradients multiply to -1 : a gradient of 2 needs $-1/2$. Parallel lines have equal gradients.

Applying it

- Plot a straight line using two checked points and label its intercept.
- Perpendicular-gradient formulae and circle equations are not part of this Foundation/Core lesson.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Foundation tier, specification section 3.

- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Gradient:** The change in y divided by the corresponding change in x .
- Gradient is change in y divided by change in x .
- Watch for: A vertical line has no finite gradient; do not force it into $y=mx+c$.

3 · Coordinate geometry and tangents · Higher

How does a path's slope become an equation?

- A path rises 6 metres over a horizontal distance of 3 metres. Its gradient connects a diagram to an equation.
- This lesson studies **gradient**: The change in y divided by the corresponding change in x .

The method

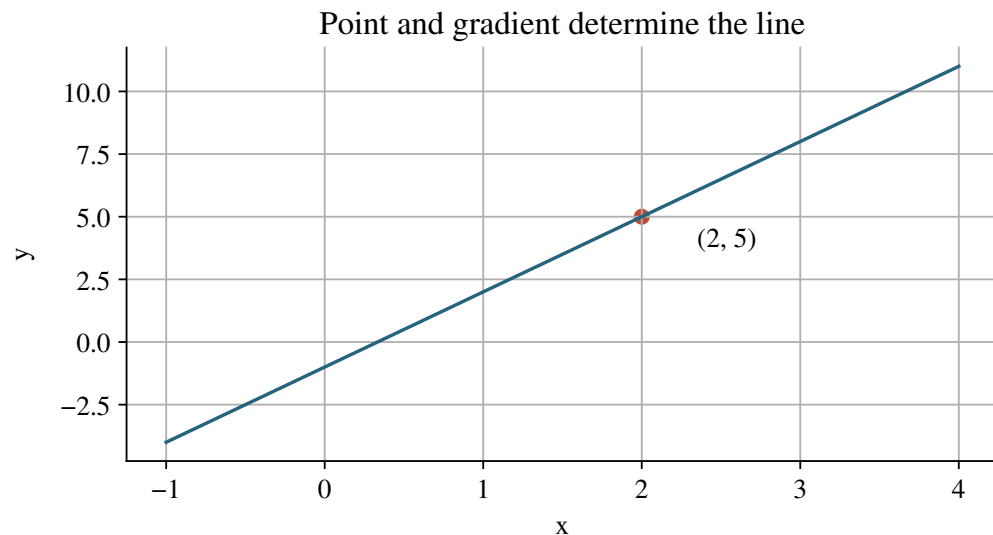
- A line through (x_1, y_1) with gradient m has $y-y_1=m(x-x_1)$.
- Parallel lines have equal gradients.
- Finite perpendicular gradients multiply to -1 .
- A circle has $(x-a)^2+(y-b)^2=r^2$.

A worked case

$$y - y_1 = m(x - x_1), \quad (x - a)^2 + (y - b)^2 = r^2$$

■ Worked example

Through $(2, 5)$ with gradient 3, $y-5=3(x-2)$, so $y=3x-1$. A perpendicular through the same point has $y-5=-(x-2)/3$. The circle $(x-2)^2+(y+1)^2=25$ has centre $(2, -1)$ and radius 5.



Where it goes wrong

- A vertical line has no finite gradient; do not force it into $y=mx+c$.
- Read the signs of a circle's centre carefully.
- The radius to a tangent is perpendicular to the tangent.

■ Watch out

Not true: Perpendicular nonvertical lines have equal gradients. Perpendicular gradients multiply to -1 : a gradient of 2 needs $-1/2$. Parallel lines have equal gradients.

Applying it

- Before solving a line-circle intersection, predict whether there are zero, one or two intersections.
- Substitution produces a quadratic whose discriminant checks the prediction.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 3.

- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Gradient:** The change in y divided by the corresponding change in x .
- A line through (x_1, y_1) with gradient m has $y-y_1=m(x-x_1)$.
- Watch for: A vertical line has no finite gradient; do not force it into $y=mx+c$.

3 · Domains, inverses and composition · Higher

Which inputs are allowed?

- A square-root model returns a real output only for some inputs. Its formula alone does not specify a complete function.
- This lesson studies **domain** : The set of allowed inputs to a function.

The method

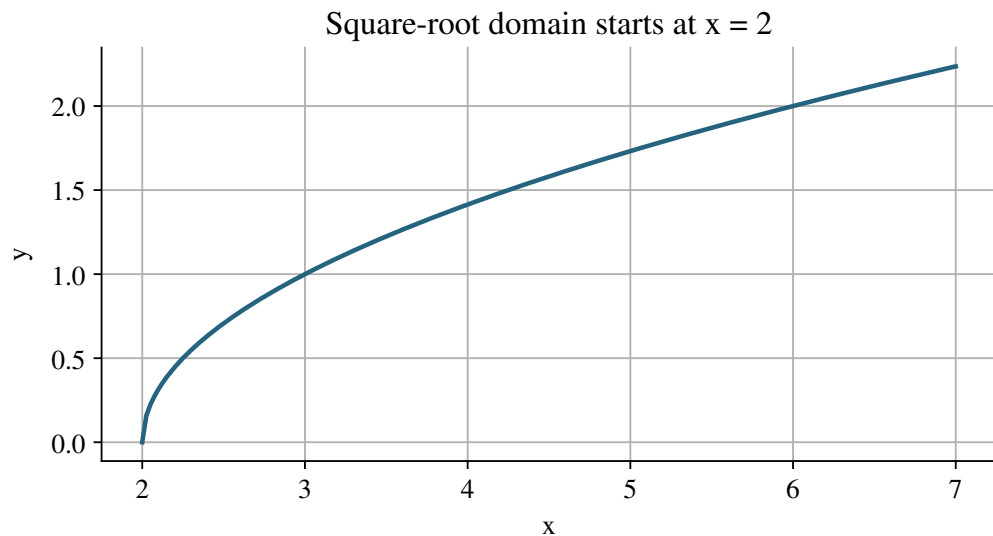
- State the domain and range.
- For an inverse, first ensure the function is one-to-one on its domain.
- Composition fg means apply g first, then f ; the intermediate output must be an allowed input to f .

A worked case

$$f(g(x)) = (f \circ g)(x), \quad f^{-1}(f(x)) = x$$

■ Worked example

For $f(x)=\sqrt{x-2}$, $x \geq 2$ and the range is $y \geq 0$. From $y=\sqrt{x-2}$, $x=y^2+2$. Thus f inverse(x)= x^2+2 with $x \geq 0$. For $g(x)=x+3$, $fg(1)=f(4)=\sqrt{2}$.



Where it goes wrong

- Squaring can introduce extraneous solutions.
- Restricting a parabola's domain is essential before claiming an inverse.
- A horizontal translation inside f has the opposite sign to the graph's movement.

■ Watch out

Not true: Every quadratic function on all real numbers has an inverse function. A quadratic on all real numbers is many-to-one ($f(1)=f(-1)$ for x^2), so its domain must be restricted first.

Applying it

- Check $f(f^{-1}(x))=x$ on the inverse domain.
- Use a sketch to test whether a horizontal line meets the original graph more than once.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 3.

- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Domain:** The set of allowed inputs to a function.
- State the domain and range.
- Watch for: squaring can introduce extraneous solutions.

3 · Derivatives and stationary points · Higher

What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
- This lesson studies **derivative**: The instantaneous rate of change, also the gradient of a tangent.

The method

- For a polynomial term ax^n with nonnegative integer n , the gradient term is $anx^{(n-1)}$.
- Add the differentiated terms.
- A stationary point has zero gradient.

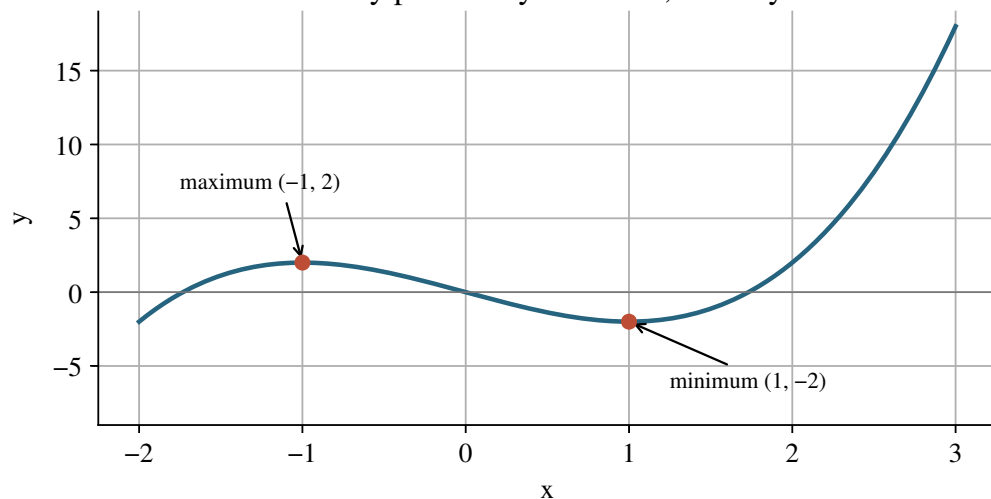
A worked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \quad f'(x) = 0$$

■ Worked example

For $y=x^3-3x$, $dy/dx=3x^2-3$. At $x=1$, the gradient is 0 and $y=-2$. The second derivative is $6x$, positive at $x=1$, so this is a local minimum. At $x=-1$, $y=2$ and the second derivative is negative, giving a local maximum.

Stationary points of $y = x^3 - 3x$, where $y' = 0$



Where it goes wrong

- A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing.
- An endpoint can also produce an extreme value on a restricted domain.

■ Watch out

Not true: Every point with zero derivative is a local maximum or minimum. A zero derivative can also be a point of inflection: $y=x^3$ at $x=0$ is neither a maximum nor a minimum.

Applying it

- This advanced-tier IGCSE lesson is limited to polynomial differentiation, tangent gradients and stationary points.
- Chain, product, quotient and implicit differentiation are excluded.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 3.

- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Derivative:** The instantaneous rate of change, also the gradient of a tangent.
- For a polynomial term ax^n with nonnegative integer n , the gradient term is $anx^{(n-1)}$.
- Watch for: A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing.

4 · Angles, lengths and area · Foundation

Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
- This lesson studies **scale factor** : The multiplier that relates corresponding lengths in similar shapes.

The method

- Use angle facts with a stated reason.
- Similar shapes have equal corresponding angles and proportional corresponding lengths.
- Areas of rectangles and triangles come from their dimensions; compound shapes can be split into simpler parts.

A worked case

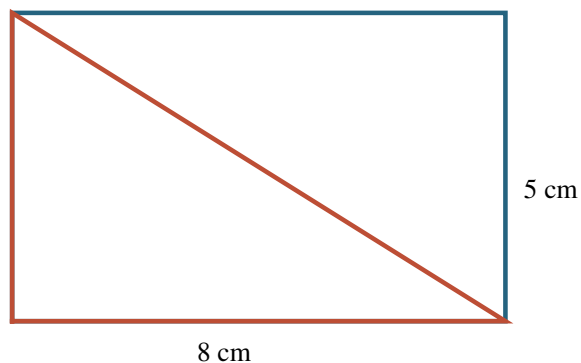
$$A_{\text{rectangle}} = LW, \quad A_{\text{triangle}} = \frac{1}{2}bh$$

■ Worked example

A rectangle of length 8 cm and width 5 cm has area $A=LW=40 \text{ cm}^2$. A triangle on the same base and height has area $A=bh/2=20 \text{ cm}^2$. For a pentagon, the interior-angle sum is $(5-2) \times 180=540^\circ$.

Same base and height, half the area

Rectangle: 40 cm²; triangle: 20 cm²



Where it goes wrong

- Equal angles alone establish similarity, not equal size.
- Use corresponding lengths in the same order.
- Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

■ Watch out

Not true: A triangle and rectangle with the same base and height have the same area. Volume scales by the cube of the length factor: doubling every length multiplies the volume by 8.

Applying it

- Use a labelled sketch and appropriate units.
- This Foundation/Core lesson does not test area/volume scale factors or advanced circle-theorem proofs.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Foundation tier, specification section 4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Scale factor:** The multiplier that relates corresponding lengths in similar shapes.
- Use angle facts with a stated reason.
- Watch for: equal angles alone establish similarity, not equal size.

4 · Angle reasoning, similarity and mensuration · Higher

Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
- This lesson studies **scale factor** : The multiplier that relates corresponding lengths in similar shapes.

The method

- For similar shapes with length scale factor k , areas scale by k^2 and volumes by k^3 .
- State angle reasons explicitly.
- A circle's tangent is perpendicular to the radius at the contact point.

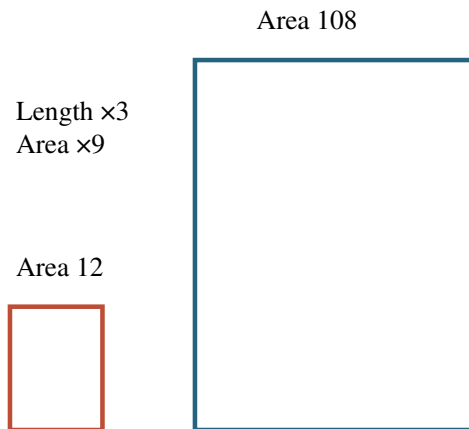
A worked case

$$\frac{A_2}{A_1} = k^2, \quad \frac{V_2}{V_1} = k^3$$

■ Worked example

If model-to-real length factor is 3, a model area of 12 cm² gives $12 \times 3^2 = 108$ cm² and a model volume of 8 cm³ gives $8 \times 3^3 = 216$ cm³. A cylinder with $r=3, h=5$ has volume $\pi r^2 h = 45\pi$.

A length scale factor is squared for area



Where it goes wrong

- Equal angles alone establish similarity, not equal size.
- Use corresponding lengths in the same order.
- Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

■ Watch out

Not true: Doubling every length of a solid doubles its volume. Volume scales by the cube of the length factor: doubling every length multiplies the volume by 8.

Applying it

- A geometric proof should name the relevant theorem, identify the equal angle or ratio, and draw the conclusion.
- A scale drawing is evidence only when the task permits measurement.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 4.

- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Scale factor:** The multiplier that relates corresponding lengths in similar shapes.
- For similar shapes with length scale factor k , areas scale by k^2 and volumes by k^3 .
- Watch for: equal angles alone establish similarity, not equal size.

4 · Right triangles and non-right triangles · Foundation

Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
- This lesson studies **hypotenuse** : The side opposite the right angle in a right-angled triangle.

The method

- Use Pythagoras in a right triangle and use sine, cosine or tangent with the sides labelled relative to the chosen angle.

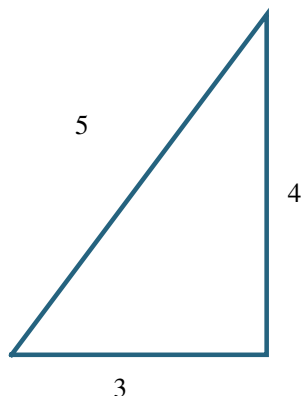
A worked case

$$a^2 + b^2 = c^2, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

■ Worked example

The ladder length is $c = \sqrt{3^2 + 4^2} = 5$ m. Its angle to the ground satisfies $\tan \theta = 4/3$, so $\theta \approx 53.1^\circ$. A right triangle with legs 6 and 8 has area $6 \times 8 / 2 = 24$.

Label the known sides before choosing a rule



Where it goes wrong

- Label sides relative to the chosen angle.
- Pythagoras needs a right angle.
- A calculator angle mode error can produce a plausible but wrong result.
- Keep unrounded values for later steps.

■ Watch out

Not true: Pythagoras applies to every triangle, including triangles without a right angle. Pythagoras needs a right angle; for other triangles use the cosine rule.

Applying it

- This Foundation/Core lesson uses right-angled triangles only.
- Sine and cosine rules for non-right triangles belong to the advanced-tier lesson.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Foundation tier, specification section 4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Hypotenuse:** The side opposite the right angle in a right-angled triangle.
- Use Pythagoras in a right triangle and use sine, cosine or tangent with the sides labelled relative to the chosen angle.
- Watch for: label sides relative to the chosen angle.

4 · Right triangles and non-right triangles · Higher

Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
- This lesson studies **hypotenuse** : The side opposite the right angle in a right-angled triangle.

The method

- In a right triangle $a^2 + b^2 = c^2$; $\sin\theta = \text{opposite/hypotenuse}$, $\cos\theta = \text{adjacent/hypotenuse}$ and $\tan\theta = \text{opposite/adjacent}$.
- For other triangles, use the sine or cosine rule, or $\text{area} = ab \sin C/2$.

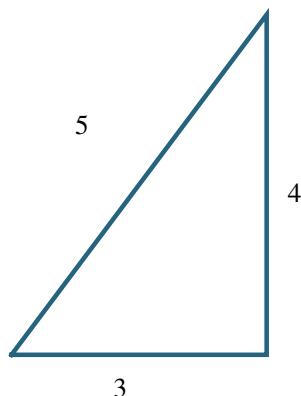
A worked case

$$a^2 + b^2 = c^2, \quad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

■ Worked example

The ladder length is $c = \sqrt{3^2 + 4^2} = 5$ m. Its angle to the ground satisfies $\tan\theta = 4/3$, so $\theta \approx 53.1^\circ$. With two sides 6 and 8 enclosing 60° , $c^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \cos 60^\circ = 52$.

Label the known sides before choosing a rule



Where it goes wrong

- Label sides relative to the chosen angle.
- Pythagoras needs a right angle.
- A calculator angle mode error can produce a plausible but wrong result.
- Keep unrounded values for later steps.

■ Watch out

Not true: Pythagoras applies to every triangle, including triangles without a right angle. Pythagoras needs a right angle; for other triangles use the cosine rule.

Applying it

- Use a plan or elevation for a three-dimensional problem before applying a triangle rule.
- Explain why the chosen triangle contains the required length or angle.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 4.

- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Hypotenuse:** The side opposite the right angle in a right-angled triangle.
- In a right triangle $a^2 + b^2 = c^2$; $\sin\theta = \text{opposite/hypotenuse}$, $\cos\theta = \text{adjacent/hypotenuse}$ and $\tan\theta = \text{opposite/adjacent}$.
- Watch for: label sides relative to the chosen angle.

4 · Circle theorems and reasoned proofs · Higher

Which angles see the same chord?

- Two observers see the same chord from the circle. Their angles are linked by a theorem rather than by the apparent size of the drawing.
- This lesson studies **cyclic quadrilateral**: A quadrilateral whose four vertices lie on one circle.

The method

- The angle at the centre is twice the angle at the circumference on the same arc.
- Angles in the same segment are equal.
- Opposite angles of a cyclic quadrilateral sum to 180° .
- A radius is perpendicular to a tangent at contact.

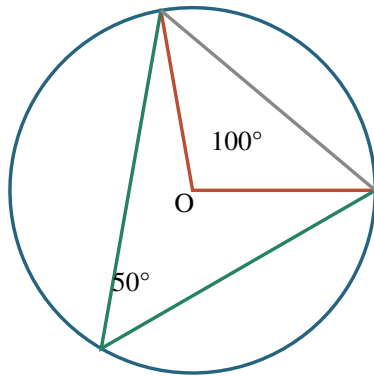
A worked case

$$\theta_{\text{centre}} = 2\theta_{\text{circumference}}$$

■ Worked example

If a central angle is 100° , the corresponding angle at the circumference is 50° . In a cyclic quadrilateral with one angle 112° , its opposite angle is $180 - 112 = 68^\circ$. A radius meeting a tangent gives 90° , even if the drawing looks oblique.

The centre angle is twice the circumference angle



Where it goes wrong

- Identify the same chord and the correct arc before using a theorem.
- Two visible right angles do not prove a quadrilateral cyclic without a valid converse argument.
- A diagram need not be to scale.

■ Watch out

Not true: Every quadrilateral has opposite angles summing to 180° . Opposite angles sum to 180° only in a cyclic quadrilateral, one whose vertices lie on a circle.

Applying it

- Write one reason alongside each angle calculation.
 - For the alternate-segment theorem, name the tangent and chord, then identify the angle in the opposite segment.
 - Use auxiliary radii only when they help the proof.
-

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Cyclic quadrilateral:** A quadrilateral whose four vertices lie on one circle.
- The angle at the centre is twice the angle at the circumference on the same arc.
- Watch for: identify the same chord and the correct arc before using a theorem.

4 · Constructions, loci and geometric conditions · Foundation

Where can both conditions hold?

- A router must be equally far from two rooms and within reach of a power point. Each condition creates a different set of possible positions.
 - This lesson studies **locus** : The set of all points satisfying a stated geometric condition.
-

The method

- Points equally distant from A and B lie on the perpendicular bisector of AB.
 - Points at fixed distance r from C lie on a circle.
 - Points equally distant from two intersecting lines lie on their angle bisectors.
-

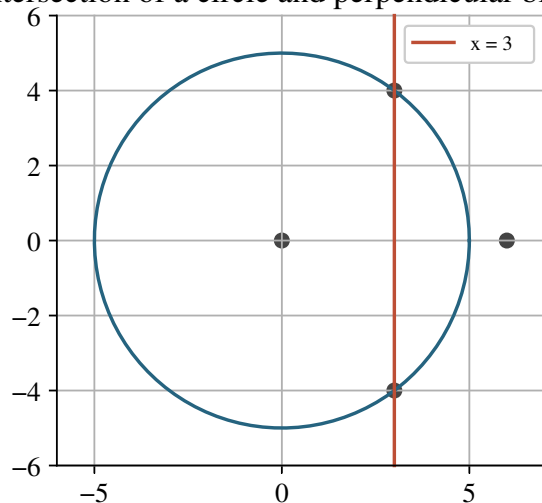
A worked case

$$x = 3, \quad x^2 + y^2 = 25$$

■ Worked example

For $A=(0,0)$ and $B=(6,0)$, the perpendicular bisector is $x=3$. Points also 5 units from A satisfy $x^2+y^2=25$. Substituting $x=3$ gives $y^2=16$, so $(3,4)$ and $(3,-4)$ satisfy both conditions.

Intersection of a circle and perpendicular bisector



Where it goes wrong

- The perpendicular bisector concerns distance to two points; the angle bisector concerns distance to two lines.
- A sketch is not a ruler-and-compass construction: preserve arcs as evidence of the method.

■ Watch out

Not true: Points equally distant from two points lie on their angle bisector. Points equidistant from two points lie on the perpendicular bisector of the segment joining them; the angle bisector is equidistant from two lines.

Applying it

- Translate each condition into a locus before finding intersections.
- For a region closer to A than B, choose the correct side of the perpendicular bisector and show whether a boundary is allowed.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Foundation tier, specification section 4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Locus:** The set of all points satisfying a stated geometric condition.
- Points equally distant from A and B lie on the perpendicular bisector of AB.
- Watch for: the perpendicular bisector concerns distance to two points; the angle bisector concerns distance to two lines.

4 · Constructions, loci and geometric conditions · Higher

Where can both conditions hold?

- A router must be equally far from two rooms and within reach of a power point. Each condition creates a different set of possible positions.
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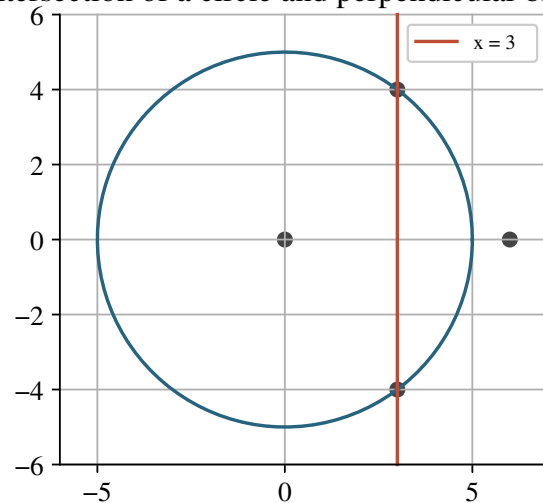
A worked case

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■ Watch out

Not true: Points equally distant from two points lie on their angle bisector. Points equidistant from two points lie on the perpendicular bisector of the segment joining them; the angle bisector is equidistant from two lines.

Applying it

- Translate each condition into a locus before finding intersections.
- For a region closer to A than B, choose the correct side of the perpendicular bisector and show whether a boundary is allowed.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Locus:** The set of all points satisfying a stated geometric condition.
- Points equally distant from A and B lie on the perpendicular bisector of AB.
- Watch for: the perpendicular bisector concerns distance to two points; the angle bisector concerns distance to two lines.

5 · Vectors and transformation geometry · Foundation

Why is displacement shorter than the walk?

- Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
- This lesson studies **resultant** : The vector sum representing the combined displacement or force.

The method

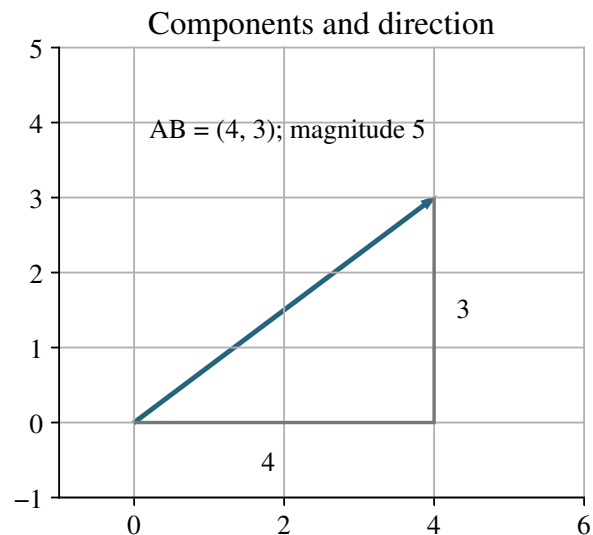
- Add corresponding vector components and subtract position vectors to find a displacement.
- A translation moves every point by the same vector; a scalar multiple changes length and possibly direction.

A worked case

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}, \quad |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

■ Worked example

With $\mathbf{a}=(4,1)$ and $\mathbf{b}=(1,3)$, $\mathbf{a}+\mathbf{b}=(5,4)$. From $A=(1,2)$ to $B=(5,5)$, displacement $\overrightarrow{AB}=(4,3)$. Its magnitude is $\sqrt{4^2+3^2}=5$.



Where it goes wrong

- The order of subtraction matters: $\vec{BA} = -\vec{AB}$.
- Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient.
- Negative enlargement reverses position about its centre.

■ Watch out

Not true: $\vec{AB}$ and $\vec{BA}$ have the same components. $\vec{BA}$ goes the opposite way to $\vec{AB}$, so $\vec{BA} = -\vec{AB}$: every component changes sign.

Applying it

- Draw arrows with direction and identify the starting and ending points.
- Scalar products, spatial line equations and advanced angle calculations are excluded from this Foundation/Core lesson.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Foundation tier, specification section 5.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Resultant:** The vector sum representing the combined displacement or force.
- Add corresponding vector components and subtract position vectors to find a displacement.
- Watch for: the order of subtraction matters: $\vec{BA} = -\vec{AB}$.

5 · Vectors and transformation geometry · Higher

Why is displacement shorter than the walk?

- Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
- This lesson studies **resultant** : The vector sum representing the combined displacement or force.

The method

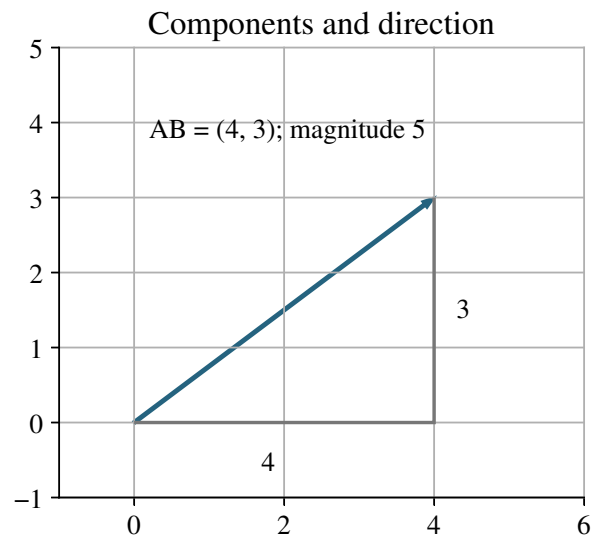
- Add corresponding vector components and subtract position vectors to find a displacement.
- A translation moves every point by the same vector; a scalar multiple changes length and possibly direction.

A worked case

$$\vec{AB} = \mathbf{b} - \mathbf{a}, \quad |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

■ Worked example

With $\mathbf{a} = (4, 1)$ and $\mathbf{b} = (1, 3)$, $\mathbf{a} + \mathbf{b} = (5, 4)$. From $A = (1, 2)$ to $B = (5, 5)$, displacement $\vec{AB} = (4, 3)$. Its magnitude is $\sqrt{4^2 + 3^2} = 5$.



Where it goes wrong

- The order of subtraction matters: $BA = -AB$.
- Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient.
- Negative enlargement reverses position about its centre.

■ Watch out

Not true: AB and BA have the same components. BA goes the opposite way to AB , so $BA = -AB$: every component changes sign.

Applying it

- Draw arrows with direction and identify the starting and ending points.
- Scalar products, spatial line equations and advanced angle calculations are excluded from this Foundation/Core lesson.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 5.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Resultant:** The vector sum representing the combined displacement or force.
- Add corresponding vector components and subtract position vectors to find a displacement.
- Watch for: the order of subtraction matters: $BA = -AB$.

5 · Reflections, rotations and enlargements · Foundation

Where is the enlargement centre?

- A logo is enlarged around a point away from the origin. Multiplying its coordinates alone puts it in the wrong place.
- This lesson studies **centre of enlargement**: The point from which each point's displacement is multiplied by a scale factor.

The method

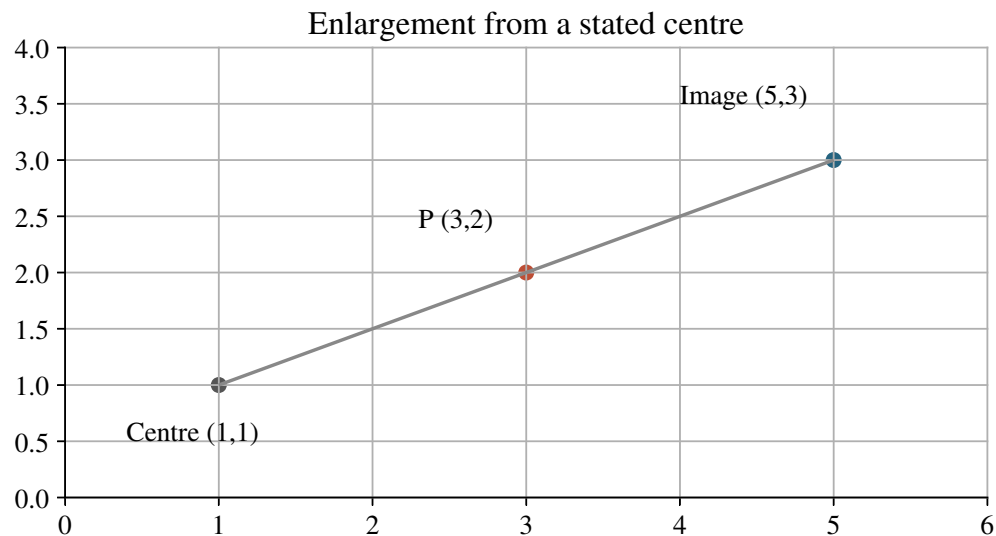
- A translation adds a vector.
- A reflection reverses signed perpendicular distance from a mirror line.
- A rotation needs a centre, angle and direction.
- For enlargement from C , use new $P = C + k(P - C)$.

A worked case

$$\mathbf{p}_{\text{image}} = \mathbf{c} + k(\mathbf{p} - \mathbf{c})$$

■ Worked example

For $C = (1,1)$, $P = (3,2)$, $k = 2$, $P - C = (2,1)$, so new $P = (1,1) + 2(2,1) = (5,3)$. Reflection of $(3,2)$ in the y -axis gives $(-3,2)$. A 90° anticlockwise rotation about the origin gives $(-2,3)$.



Where it goes wrong

- A rotation needs its centre and direction, not just an angle.
- A negative enlargement factor places the image on the opposite side of the centre.
- A translation does not change orientation or size.

■ Watch out

Not true: Every enlargement is centred at the origin unless its scale factor is negative. An enlargement can have any centre; a negative scale factor puts the image on the other side of the centre.

Applying it

- Describe a transformation completely before constructing the image.
- Check corresponding distances and angles.
- For combined transformations, apply them in the stated order; they usually do not commute.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Foundation tier, specification section 5.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Centre of enlargement:** The point from which each point's displacement is multiplied by a scale factor.
- A translation adds a vector.
- Watch for: A rotation needs its centre and direction, not just an angle.

5 · Reflections, rotations and enlargements · Higher

Where is the enlargement centre?

- A logo is enlarged around a point away from the origin. Multiplying its coordinates alone puts it in the wrong place.
- This lesson studies **centre of enlargement** : The point from which each point's displacement is multiplied by a scale factor.

The method

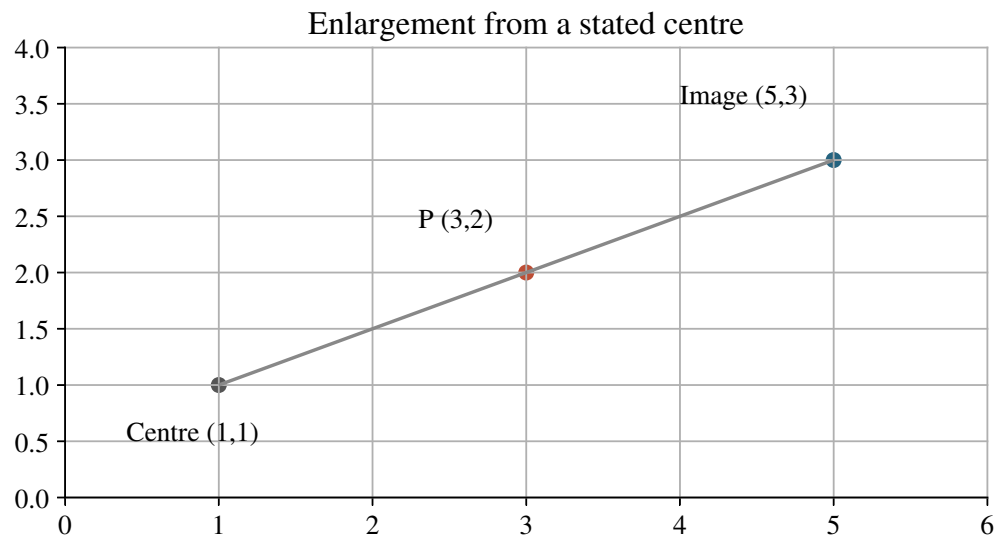
- A translation adds a vector.
- A reflection reverses signed perpendicular distance from a mirror line.
- A rotation needs a centre, angle and direction.
- For enlargement from C, use new $P = C + k(P - C)$.

A worked case

$$\mathbf{p}_{\text{image}} = \mathbf{c} + k(\mathbf{p} - \mathbf{c})$$

■ Worked example

For $C=(1,1), P=(3,2), k=2$, $P-C=(2,1)$, so new $P=(1,1)+2(2,1)=(5,3)$. Reflection of $(3,2)$ in the y-axis gives $(-3,2)$. A 90° anticlockwise rotation about the origin gives $(-2,3)$.



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Applying it

- Describe a transformation completely before constructing the image.
- Check corresponding distances and angles.
- For combined transformations, apply them in the stated order; they usually do not commute.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 5.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Centre of enlargement:** The point from which each point's displacement is multiplied by a scale factor.
- A translation adds a vector.
- Watch for: A rotation needs its centre and direction, not just an angle.

6 · Centre, spread and data displays · Foundation

Can one average tell the whole story?

- Two groups have the same median but different spread. One summary cannot describe both location and consistency.
- This lesson studies **range** : The maximum observed value minus the minimum observed value.

The method

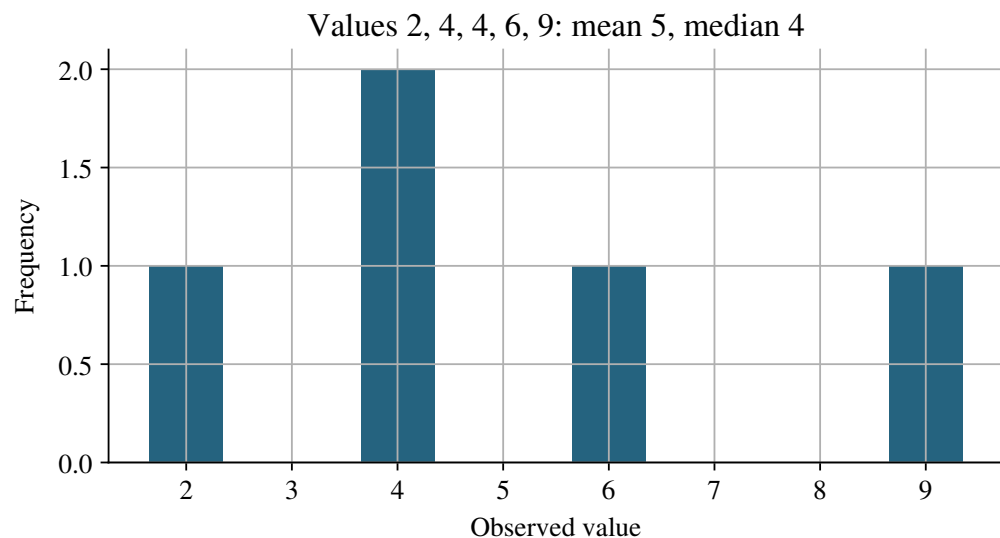
- The mean is total divided by count.
- The median is the central value after sorting.
- The range is maximum minus minimum.
- Use frequency tables, bar charts and suitable comparisons.

A worked case

$$\bar{x} = \frac{\sum x_i}{n}$$

■ Worked example

For 2,4,4,6,9, total=25 and count=5, so mean=5. The central value is 4, so median=4. The range is 9-2=7. Explain both a typical value and the spread.



Where it goes wrong

- The tallest histogram bar need not contain the most observations.
- A grouped mean is an estimate.
- Correlation does not prove causation, and extrapolation extends beyond the observed range.

■ Watch out

Not true: The mean of a list must be one of its observed values. With unequal class widths the height is frequency density; frequency is the area of the bar.

Applying it

- A bar chart uses separate bars for categories.
- Unequal-class-width histograms and formal density calculations are outside this Foundation/Core lesson.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Foundation tier, specification section 6.

- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Range:** The maximum observed value minus the minimum observed value.
- The mean is total divided by count.
- Watch for: the tallest histogram bar need not contain the most observations.

6 · Data summaries, histograms and interpretation · Higher

Can one average tell the whole story?

- Two groups have the same median but different spread. One summary cannot describe both location and consistency.
- This lesson studies **frequency density** : Frequency divided by class width, used as histogram height.

The method

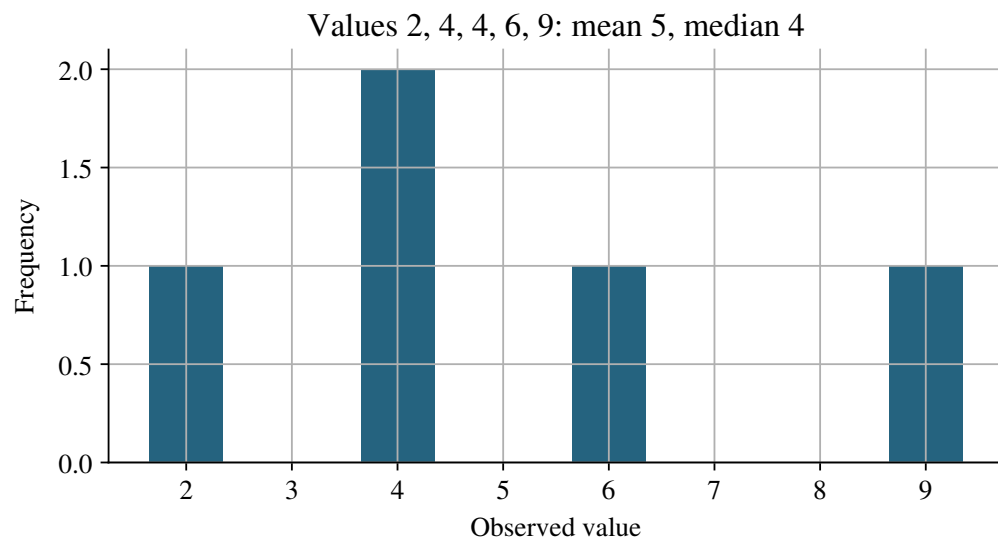
- Compare an appropriate average and spread in context.
- A histogram uses area for frequency, so height=frequency/class width.
- Grouped estimates assume representative values within intervals.

A worked case

$$\bar{x} = \frac{\sum x_i}{n}, \quad \text{density} = \frac{\text{frequency}}{\text{class width}}$$

■ Worked example

A class from 10 to 20 with frequency 30 has density 30/10=3. A class from 20 to 40 with frequency 20 has density 20/20=1. Its wider bar must not be mistaken for a larger density. For values 2,4,4,6,9, the median is 4 and mean is 5.



Where it goes wrong

- The tallest histogram bar need not contain the most observations.
- A grouped mean is an estimate.
- Correlation does not prove causation, and extrapolation extends beyond the observed range.

■ Watch out

Not true: A histogram bar's height equals its frequency, even with unequal class widths. With unequal class widths the height is frequency density; frequency is the area of the bar.

Applying it

- Choose a display that fits the data type.
- Give both a numerical comparison and what it means for the population; do not infer more precision than the sample supports.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 6.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Frequency density:** Frequency divided by class width, used as histogram height.
- Compare an appropriate average and spread in context.
- Watch for: the tallest histogram bar need not contain the most observations.

6 · Probability trees and outcomes · Foundation

What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability** : The probability of an event after restricting the sample space to a stated condition.

The method

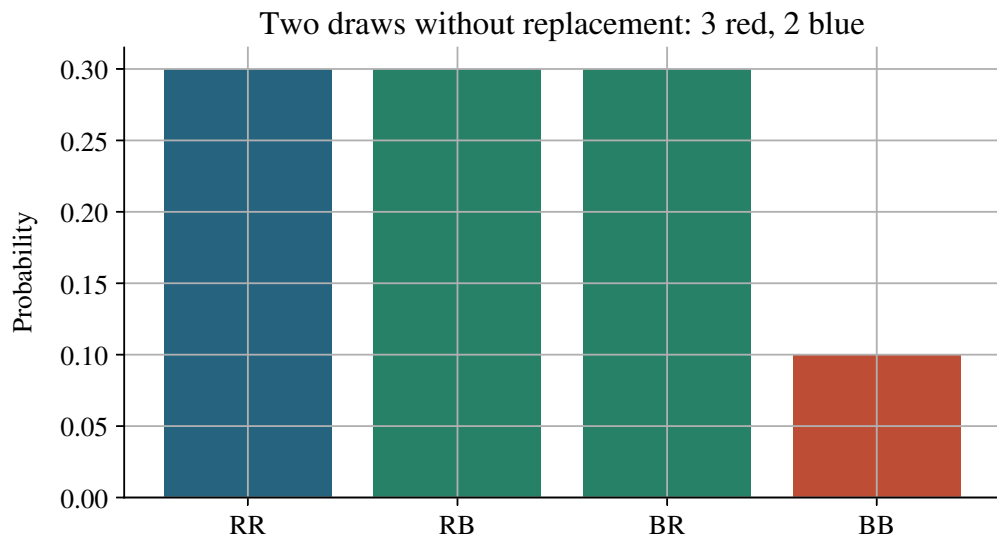
- A probability lies between 0 and 1.
- Exhaustive, mutually exclusive outcomes have probabilities summing to 1.
- Multiply successive branch probabilities and add separate routes to an outcome.

A worked case

$$P(RR) = P(R_1)P(R_2 | R_1)$$

■ Worked example

A bag contains 3 red and 2 blue counters. With replacement, $P(\text{two red}) = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25} = 0.36$. Without replacement, the red-red branch is $\frac{3}{5} \times \frac{2}{4} = 0.3$. Label each branch before multiplying.



Where it goes wrong

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.
- Two disjoint events with positive probability are not independent.

■ Watch out

Not true: Mutually exclusive events with positive probabilities must be independent. Mutually exclusive events with positive probabilities are dependent: if one happens, the other cannot, so $P(A \text{ and } B) = 0 \neq P(A)P(B)$.

Applying it

- Use a frequency table or a simple tree before calculating.
- Formal conditional probability formulae are outside this Foundation/Core support lesson.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Foundation tier, specification section 6.

- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Conditional probability:** The probability of an event after restricting the sample space to a stated condition.
- A probability lies between 0 and 1.
- Watch for: mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.

6 · Probability, trees and conditional reasoning · Higher

What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability** : The probability of an event after restricting the sample space to a stated condition.

The method

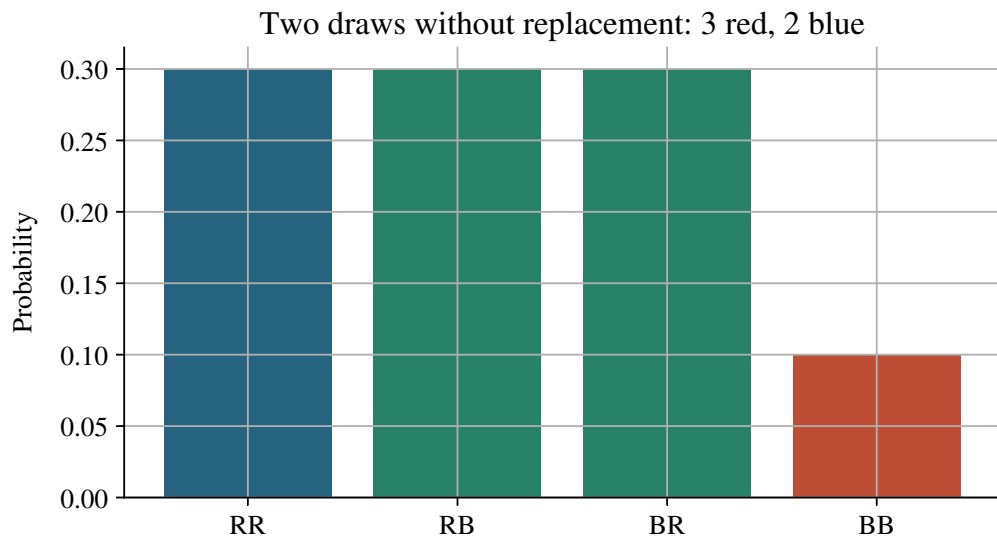
- Multiply along a tree branch and add disjoint branches.
- With replacement, the composition stays fixed.
- Conditional probability is $P(A \text{ given } B) = P(A \cap B) / P(B)$, for $P(B) > 0$.

A worked case

$$P(A | B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

■ Worked example

Without replacement, $P(\text{two red}) = 3/5 \times 2/4 = 3/10$. $P(\text{one of each}) = 3/5 \times 2/4 + 2/5 \times 3/4 = 3/5$. If $P(A \cap B) = 0.12$ and $P(B) = 0.3$, $P(A \text{ given } B) = 0.4$.



Where it goes wrong

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.
- Two disjoint events with positive probability are not independent.

■ Watch out

Not true: Mutually exclusive events with positive probabilities must be independent. Mutually exclusive events with positive probabilities are dependent: if one happens, the other cannot, so $P(A \text{ and } B) = 0 \neq P(A)P(B)$.

Applying it

- A two-way table makes the restricted denominator visible.
- Before using a product $P(A)P(B)$, justify independence from the context or the supplied information.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 6.

- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Conditional probability:** The probability of an event after restricting the sample space to a stated condition.
- Multiply along a tree branch and add disjoint branches.
- Watch for: mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.

6 · Cumulative frequency and box plots · Higher

Which route is more consistent?

- Two bus routes have similar typical times but different reliability. Quartiles show the middle half of the journeys.
- This lesson studies **interquartile range** : The difference between the upper and lower quartiles.

The method

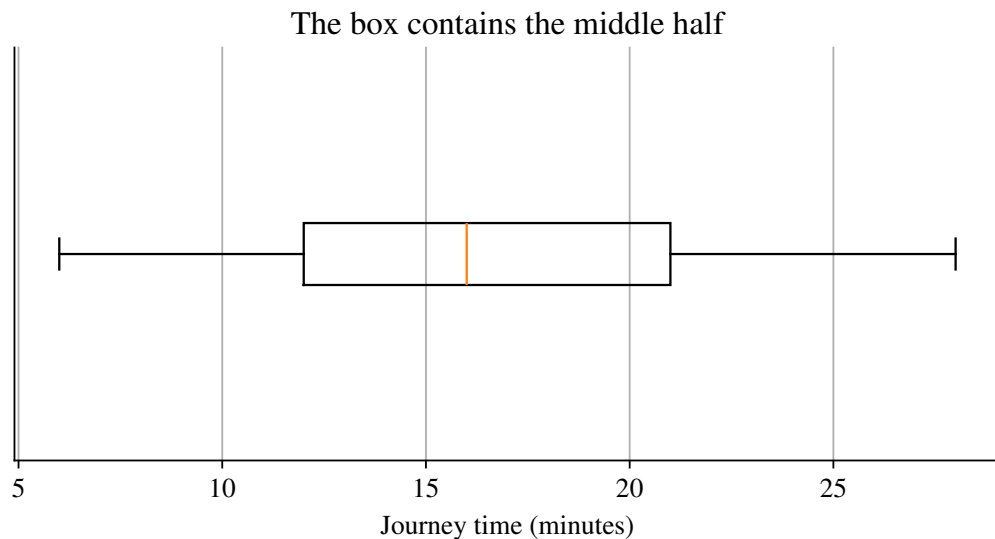
- A cumulative frequency counts observations below successive class boundaries.
- Read quartiles at one quarter, one half and three quarters of the total frequency.
- A box plot represents minimum, lower quartile, median, upper quartile and maximum.

A worked case

$$\text{IQR} = Q_3 - Q_1$$

■ Worked example

For 80 observations, read Q_1 at cumulative frequency 20, median at 40 and Q_3 at 60. If $Q_1=12, Q_3=21$, then $\text{IQR}=9$. Compare the medians for typical journey time and the IQRs for consistency.



■ Key idea

- **Interquartile range:** The difference between the upper and lower quartiles.
- A cumulative frequency counts observations below successive class boundaries.
- Watch for: plot against class boundaries rather than midpoints.

Where it goes wrong

- Plot against class boundaries rather than midpoints.
- Grouped quartiles are estimates.
- The range is sensitive to extremes; the IQR describes only the middle half.

■ Watch out

Not true: The interquartile range equals the maximum minus the minimum. The interquartile range is $Q3 - Q1$; maximum minus minimum is the range.

Applying it

- Explain a comparison in the context of the measured quantity.
- An outlier rule may use $Q1 - 1.5IQR$ and $Q3 + 1.5IQR$; use the rule specified in the task rather than assuming every graph follows it.

In your course

- Pearson Edexcel International GCSE Mathematics A (4MA1), Higher tier, specification section 6.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.