Further Pure Mathematics 2

Edexcel IAL FP2 (WFM02/01); optional pure unit beyond the P1–P4 + FP1 core

1. Unit FP2 at a glance

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WFM02/01: 1 hour 30 minutes, 75 marks; P1–P4 and FP1 assumed and testable. Must-know: the modulus definition and $|z_1 z_2| = |z_1||z_2|$, partial fractions, integration by parts and substitution, and separation of variables.

2. Inequalities involving the modulus sign

Modulus inequalities

Case split from $|f| = f$ where $f \geq 0$ and $-f$ where $f < 0$, or square $|A| > |B|$ into $A^2 > B^2$. Never divide by an unfixed sign: split on it. A dead branch is a result to report. Demand pattern: $x - 2 > \frac{15x}{|x|+4}$, then the modulus-of-both-sides rerun.

3. The method of differences

The method of differences

Split the summand so consecutive terms cancel: $\frac{1}{(r+1)(r+2)} = \frac{1}{r+1} - \frac{1}{r+2}$ telescopes to $\frac{n}{2(n+2)}$. Display the $r = 1, 2, n-1, n$ lines; close follow-ups by algebra, not tables.

4. Euler's relation and de Moivre's theorem

Euler and de Moivre

$e^{i\theta} = \cos\theta + i\sin\theta$, with $\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$ and $\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$. De Moivre: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, provable by induction for integers.
Binomial-expand, then substitute $c^2 = 1 - s^2$: $\sin 5\theta = 16s^5 - 20s^3 + 5s$.

5. Loci, regions and z→w transformations

Loci and Möbius maps

Circles $|z-a|=b$, bisectors $|z-a|=|z-b|$, Apollonius circles for $k \neq 1$, argument half-lines. For $w=\frac{az+b}{cz+d}$: make z the subject, then equate moduli or arguments. Circle through the pole maps to a line: $|z|=1$ under $w=\frac{9i-iz}{1+z}$ gives $20v=80$, so $v=4$.

6. First order differential equations

First order ODEs

For $\frac{dy}{dx} + Py = Q$, multiply by the integrating factor $e^{\int P\,dx}$ and integrate once. Given substitutions like $z = 1/y^2$: chain rule, rewrite, solve, substitute back. y^2 determined means $y = \pm$: both signs are answers.

7. Second order differential equations

Second order ODEs

Auxiliary $am^2 + bm + c = 0$: distinct roots $Ae^{m_1 x} + Be^{m_2 x}$, repeated $(A + Bx)e^{mx}$, complex $e^{px}(A\cos qx + B\sin qx)$. Trial PI by right-hand side; if the trial solves the homogeneous equation, multiply by x: $4y'' - 11y' - 3y = 78e^{3x}$ needs $\lambda x e^{3x}$ with $\lambda = 6$.

8. Maclaurin and Taylor series

Maclaurin and Taylor

Maclaurin: $\sum f^{(n)}(0)x^n/n!$; Taylor about a replaces x by $(x-a)$. For ODEs, differentiate the equation to climb the ladder: $y'' - 4y' - 5y = e^{2x/9}$ with $y(3) = 2$, $y'(3) = -1$ gives $y''(3) = 6 + e^{2/3}$, then $y'''(3) = 19 + \frac{38}{9}e^{2/3}$.

9. Polar coordinates and areas

Polar coordinates

$x = r\cos\theta$, $y = r\sin\theta$, $r^2 = x^2 + y^2$: $r = 4\cos\theta$ is $(x-2)^2 + y^2 = 4$. Staples: circle $2a\cos\theta$, cardioid $a(1+\cos\theta)$, lemniscate $r^2 = a^2\cos 2\theta$. Area $= \frac{1}{2}\int r^2\, d\theta$; tangent parallel to the initial line where $\frac{dr}{d\theta}\sin\theta + r\cos\theta = 0$.

Check yourself

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Solve a modulus inequality with $|x|$ in the denominator; telescope a series to closed form; expand a multiple-angle identity and find all fifth roots; make z the subject of a Möbius map and equate moduli; integrating factor and substitution first order ODEs; second order with resonance; Taylor series from the equation; polar area exactly.