Further Pure Mathematics 1

Edexcel IAL FP1 (WFM01/01); compulsory for IAS/IAL Further Mathematics and Pure Mathematics

1. Unit FP1 at a glance

Unit FP1 at a glance

WFM01/01: 1 hour 30 minutes, 75 marks; P1–P2 assumed and testable. Must-know: $\alpha + \beta = -b/a$, $\alpha\beta = c/a$, $\sum r = \frac{1}{2}n(n+1)$, the 2×2 inverse, and the standard transformation matrices.

2. Complex numbers

Complex numbers

$z = a + bi$ with modulus $\sqrt{a^2 + b^2}$, conjugate $a - bi$, and polar $r(\cos\theta + i\sin\theta)$.
Quotients: multiply by the conjugate of the denominator. $|z_1 z_2| = |z_1||z_2|$.

3. Complex roots of polynomials

Complex roots of polynomials

Real coefficients: z_1 a root forces $\overline{z}_1$ a root, giving the factor $z^2 - 2\mathrm{Re}(z_1)z + |z_1|^2$.
Compare coefficients or divide to finish a cubic or quartic.

4. Roots of quadratic equations

Roots of quadratic equations

$\alpha + \beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$. Symmetric expressions: $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$; $\alpha^3 + \beta^3 \equiv (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$. New equation: $x^2 - (\text{sum})x + \text{product}$.

5. Numerical solution of equations

Numerical solution of equations

Sign change plus continuity brackets a root. Bisection halves; interpolation takes the chord zero $a - \frac{(b-a)f(a)}{f(b)-f(a)}$; Newton–Raphson iterates $x_{n+1} = x_n - f(x_n)/f'(x_n)$. Show every substitution.

6. Parabola and rectangular hyperbola

Parabola and rectangular hyperbola

$y^2 = 4ax$: parametric $(at^2, 2at)$, gradient $1/t$. $xy = c^2$: parametric $(ct, c/t)$, gradient $-1/t^2$. Focus $(a, 0)$, directrix $x = -a$, and $PF = PM$ everywhere.

7. Matrix algebra

Matrix algebra

$\det \mathbf{M} = ad - bc$; singular when zero; $\mathbf{M}^{-1} = \frac{1}{ad-bc}$ times the swapped-sign matrix; $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$. Matrix equations: compare all four entries.

8. Transformations using matrices

Transformations using matrices

Columns are the images of $\mathbf{i}$ and $\mathbf{j}$. Reflections in the axes and $y = \pm x$, the rotation matrix in $\cos\theta$ and $\sin\theta$, stretches, enlargements. $\mathbf{AB}$: apply $\mathbf{B}$ first. $|\det|$ is the area scale factor.

9. Series

Series

$\sum_{r=1}^{2n}(2r^2-1) = \frac{4}{3}n(n+1)(4n-1)$ by substituting $k = 2n$ into $\sum r^2 = \frac{1}{6}k(k+1)(2k+1)$ and subtracting $2n$.

10. Proof by mathematical induction

Proof by mathematical induction

Verify $n = 1$; assume $n = k$; add the $(k+1)$th term and match the target; close with the induction sentence. Proved example: $\sum r(3r-2)^2 = \frac{1}{4}n^2(n+1)(9n-7)$.

Check yourself

Check yourself

Conjugate quotient in $a + bi$; quartic roots from one complex root; new quadratic from the root sum and product; bisection with all values shown; normal to the rectangular hyperbola; matrix equation by entries; transformation described fully; induction closed properly.