Solutions

2.1 $\frac{dy}{dx} = \frac{3}{2}x^{1/2}$, so with $u = 1 + \frac{9}{4}x$, $du = \frac{9}{4}dx$: $s = \displaystyle\int_0^4 \sqrt{1+\tfrac{9}{4}x}\,dx = \tfrac{4}{9}\left[\tfrac{2}{3}u^{3/2}\right]_1^{10} = \tfrac{8}{27}\left(10^{3/2}-1\right)$.

2.2 $\frac{dy}{dx} = \sinh x$ and $1 + \sinh^2 x = \cosh^2 x$, so

$$S = 2\pi \int_0^{\ln 2} \cosh^2 x\,dx = 2\pi\left[\tfrac{1}{2}\sinh x\cosh x + \tfrac{x}{2}\right]_0^{\ln 2}$$

(from $\cosh^2 x = \frac{1}{2}(\cosh 2x + 1)$). With $\sinh(\ln 2) = \frac{3}{4}$, $\cosh(\ln 2) = \frac{5}{4}$: $S = 2\pi\left(\frac{15}{32} + \frac{1}{2}\ln 2\right)$, i.e. $\frac{15\pi}{16} + \pi\ln 2$.

2.3 (a) Parts with $u = \sin^{n-1}x$, $dv = \sin x\,dx$: $I_n = (n-1)\displaystyle\int_0^{\pi/2} \cos^2 x \sin^{n-2}x\,dx = (n-1)(I_{n-2} - I_n)$ (the boundary term vanishes at both limits), so $nI_n = (n-1)I_{n-2}$. (b) $I_0 = \frac{\pi}{2}$, $I_2 = \frac{1}{2}I_0 = \frac{\pi}{4}$, $I_4 = \frac{3}{4}I_2 = \frac{3\pi}{16}$.

3.1 (a) $v = \frac{2}{3}\sqrt{3x-2}$ (check by differentiating). Parts:

$$I_n = \left[\tfrac{2}{3}x^n\sqrt{3x-2}\right]_1^6 - \tfrac{2n}{3}\int_1^6 x^{n-1}\sqrt{3x-2}\,dx$$

The boundary term: $\frac{2}{3}\left(6^n\cdot 4 - 1\right) = \frac{8\cdot 6^n - 2}{3}$. The integral: $\displaystyle\int_1^6 \frac{x^{n-1}(3x-2)}{\sqrt{3x-2}}dx = 3I_n - 2I_{n-1}$. So $I_n = \frac{8\cdot 6^n-2}{3} - \frac{2n}{3}(3I_n - 2I_{n-1})$, which rearranges to $(3+6n)I_n = 4nI_{n-1} + 8\cdot 6^n - 2$. (b) $I_0 = \frac{2}{3}[\sqrt{3x-2}]_1^6 = \frac{2}{3}(4-1) = 2$; $n=1$: $9I_1 = 4(2) + 48 - 2 = 54$, so $I_1 = 6$; $n=2$: $15I_2 = 8(6) + 8(36) - 2 = 334$, so $I_2 = \frac{334}{15}$; $n=3$: $21I_3 = 12\cdot\frac{334}{15} + 8(216) - 2 = \frac{1336}{5} + 1726 = \frac{9966}{5}$, so $I_3 = \frac{9966}{105} = \frac{3322}{35}$.

3.2 $\frac{dy}{dx} = -2\sin 2x$, so $\sqrt{1+(\frac{dy}{dx})^2} = \sqrt{1+4\sin^2 2x}$ and $S = 2\pi\displaystyle\int y\sqrt{1+\left(\tfrac{dy}{dx}\right)^2}dx = 2\pi\int_0^{\pi/4}\cos 2x\sqrt{1+4\sin^2 2x}\,dx$ (no modulus needed: $\cos 2x \geq 0$ on $[0, \frac{\pi}{4}]$).

Local review guidance: parts orientation chosen so the leftover integral re-expresses in I_n, I_{n-1}; boundary values at both limits; surface integrals carry the y factor and the surd from $\frac{dy}{dx}$, with exact evaluations. This is authored classroom guidance.

3.3

(a)

- arc length, $s = \displaystyle\int \sqrt{1+\left(\tfrac{dy}{dx}\right)^2}\,dx$, with $\dfrac{dy}{dx} = \sinh x$ and $1 + \sinh^2 x = \cosh^2 x$:

$$s = \int_{-\ln 3}^{\ln 3} \cosh x\,dx = \Big[\sinh x\Big]_{-\ln 3}^{\ln 3} = 2\sinh(\ln 3)$$

- $\sinh(\ln 3) = \frac{1}{2}\left(3 - \frac{1}{3}\right) = \frac{4}{3}$, so $s = \mathbf{\frac{8}{3}}$ m

(b)

- surface area, $S = 2\pi\displaystyle\int y\sqrt{1+\left(\tfrac{dy}{dx}\right)^2}\,dx = 2\pi\int_0^{\ln 3}\cosh^2 x\,dx$
- use $\cosh^2 x = \frac{1}{2}(1+\cosh 2x)$:

$$S = \pi\int_0^{\ln 3}(1+\cosh 2x)\,dx = \pi\Big[x + \tfrac{1}{2}\sinh 2x\Big]_0^{\ln 3}$$

- $\sinh(2\ln 3) = \frac{1}{2}\left(9 - \frac{1}{9}\right) = \frac{40}{9}$:

$$S = \pi\left(\ln 3 + \tfrac{1}{2}\cdot\tfrac{40}{9}\right) = \pi\left(\ln \mathbf{3} + \mathbf{\tfrac{20}{9}}\right)\ \text{m}^2$$

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