

Solutions

2.1 $\frac{dy}{dx} = \frac{3}{2}x^{1/2}$, so with $u = 1 + \frac{9}{4}x$, $du = \frac{9}{4}dx$: $s = \int_0^4 \sqrt{1 + \frac{9}{4}x} dx = \frac{4}{9} \left[\frac{2}{3} u^{3/2} \right]_1^{10} = \frac{8}{27} (10^{3/2} - 1)$.

2.2 $\frac{dy}{dx} = \sinh x$ and $1 + \sinh^2 x = \cosh^2 x$, so

$$S = 2\pi \int_0^{\ln 2} \cosh^2 x dx = 2\pi \left[\frac{1}{2} \sinh x \cosh x + \frac{x}{2} \right]_0^{\ln 2}$$

(from $\cosh^2 x = \frac{1}{2}(\cosh 2x + 1)$). With $\sinh(\ln 2) = \frac{3}{4}$, $\cosh(\ln 2) = \frac{5}{4}$: $S = 2\pi \left(\frac{15}{32} + \frac{1}{2} \ln 2 \right)$, i.e. $\frac{15\pi}{16} + \pi \ln 2$.

2.3 (a) Parts with $u = \sin^{n-1} x$, $dv = \sin x dx$: $I_n = (n-1) \int_0^{\pi/2} \cos^2 x \sin^{n-2} x dx = (n-1)(I_{n-2} - I_n)$ (the boundary term vanishes at both limits), so $nI_n = (n-1)I_{n-2}$. (b) $I_0 = \frac{\pi}{2}$, $I_2 = \frac{1}{2}I_0 = \frac{\pi}{4}$, $I_4 = \frac{3}{4}I_2 = \frac{3\pi}{16}$.

3.1 (a) $v = \frac{2}{3}\sqrt{3x-2}$ (check by differentiating). Parts:

$$I_n = \left[\frac{2}{3} x^n \sqrt{3x-2} \right]_1^6 - \frac{2n}{3} \int_1^6 x^{n-1} \sqrt{3x-2} dx$$

The boundary term: $\frac{2}{3}(6^n \cdot 4 - 1) = \frac{8 \cdot 6^n - 2}{3}$. The integral: $\int_1^6 \frac{x^{n-1}(3x-2)}{\sqrt{3x-2}} dx = 3I_n - 2I_{n-1}$. So $I_n = \frac{8 \cdot 6^n - 2}{3} - \frac{2n}{3}(3I_n - 2I_{n-1})$, which rearranges to $(3+6n)I_n = 4nI_{n-1} + 8 \cdot 6^n - 2$. (b) $I_0 = \frac{2}{3}[\sqrt{3x-2}]_1^6 = \frac{2}{3}(4-1) = 2$; $n=1$: $9I_1 = 4(2) + 48 - 2 = 54$, so $I_1 = 6$; $n=2$: $15I_2 = 8(6) + 8(36) - 2 = 334$, so $I_2 = \frac{334}{15}$; $n=3$: $21I_3 = 12 \cdot \frac{334}{15} + 8(216) - 2 = \frac{1336}{5} + 1726 = \frac{9966}{5}$, so $I_3 = \frac{9966}{105} = \frac{3322}{35}$.

3.2 $\frac{dy}{dx} = -2 \sin 2x$, so $\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + 4 \sin^2 2x}$ and $S = 2\pi \int y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = 2\pi \int_0^{\pi/4} \cos 2x \sqrt{1 + 4 \sin^2 2x} dx$ (no modulus needed: $\cos 2x \geq 0$ on $[0, \frac{\pi}{4}]$).

Local review guidance: parts orientation chosen so the leftover integral re-expresses in I_n, I_{n-1} ; boundary values at both limits; surface integrals carry the y factor and the surd from $\frac{dy}{dx}$, with exact evaluations. This is authored classroom guidance.