

Solutions

2.1 Circle, centre $2i$ (the point $(0, 2)$), radius 3.

2.2 Perpendicular bisector of the points 3 and -1 : the vertical line $\operatorname{Re} z = 1$.

2.3 With $z = x + iy$: $(x-2)^2 + y^2 = 4[(x+1)^2 + y^2]$, so $x^2 - 4x + 4 + y^2 = 4x^2 + 8x + 4 + 4y^2$, i.e. $3x^2 + 12x + 3y^2 = 0$, so $x^2 + 4x + y^2 = 0$, i.e. $(x+2)^2 + y^2 = 4$: circle, centre $(-2, 0)$, radius 2. (Check: $z = 0$ gives $|-2| = 2|1|$ ✓ and 0 lies on it.)

2.4 $\arg\left(\frac{z-1}{z+1}\right) = \arg(z-1) - \arg(z+1)$ is the angle at z subtended by the segment from -1 to 1 . Equal to $\frac{\pi}{2}$: the semicircle $x^2 + y^2 = 1$, $y > 0$, excluding the endpoints ± 1 .

3.1 $w = \frac{1}{z} \Rightarrow z = \frac{1}{w}$. Then $|\frac{1}{w} - 2| = 2$ gives $|1 - 2w| = 2|w|$. Squaring: $1 - 2w - 2\bar{w} + 4|w|^2 = 4|w|^2$, so $w + \bar{w} = \frac{1}{2}$, i.e. $\operatorname{Re} w = \frac{1}{4}$ – a vertical line in the w -plane, $\frac{1}{4}$ unit to the right of the imaginary axis. (Check: $z = 4$ is on the circle, and $z = 4 \mapsto w = \frac{1}{4}$ ✓.)

3.2 (a) Unit circle, centre the origin. (b) $z = \frac{9i - w}{w + i}$, so $|9i - w| = |w + i| \cdot |z| = |w + i|$. With $w = u + iv$: $9i - w = -u + i(9 - v)$, so $u^2 + (9 - v)^2 = u^2 + (v + 1)^2$, giving $81 - 18v = 2v + 1$, i.e. $20v = 80$: $v = 4$. (c) $z = i$: $w = \frac{9i - i^2}{1 + i} = \frac{1 + 9i}{1 + i} = \frac{(1 + 9i)(1 - i)}{2} = \frac{10 + 8i}{2} = 5 + 4i$, whose imaginary part is 4, so it lies on l ✓.

Local review guidance: z made the subject before any modulus work; modulus equation expanded in u, v ; line/circle description with geometry named (bisector, centre, radius). This is authored classroom guidance.