Solutions

2.1 $f(1) = -1 < 0$, $f(2) = 7 > 0$, continuous $\Rightarrow$ root in $[1, 2]$. $f(1.5) = 1.875 > 0 \Rightarrow [1, 1.5]$; $f(1.25) = 0.203125 > 0 \Rightarrow [1.25, 1.5]$ (width 0.25).

2.2 The chord from $(2, -0.5)$ to $(3, 0.3\dot{3})$ crosses zero at $x \approx 2 + \dfrac{(3-2) \times 0.5}{0.3\dot{3} + 0.5} = 2 + \dfrac{0.5}{0.8\dot{3}} = 2.60$ (3 s.f.). (Check: $h(2.6) = 2.6 + 0.385 - 3 = -0.015$; true root $\frac{3+\sqrt{5}}{2} = 2.618$.)

2.3 $f'(x) = 3x^2 - 3$. $f(2) = 1$, $f'(2) = 9$: $x_1 = 2 - \frac{1}{9} = 1.8\dot{8}$. $f(1.8\dot{8}) = 6.740 - 5.667 - 1 = 0.073$; $f'(1.8\dot{8}) = 7.70$: $x_2 = 1.8\dot{8} - \frac{0.073}{7.70} = 1.8794$ (true root $1.87939\ldots$).

2.4 A bisection locates an interval, never an exact value – the root is only known to lie in $(1.6, 1.8)$ after one halving; and the midpoint 1.7 is an estimate whose accuracy is unquantified without further iterations or a stated tolerance.

3.1 (a) $p(1) = -3 < 0$, $p(2) = 9 > 0$, p continuous $\Rightarrow$ root $\gamma \in [1, 2]$. (b) $p(1.5) = 5.0625 - 5.5 = -0.4375 < 0 \Rightarrow [1.5, 2]$; $p(1.75) = 9.3789 - 6.25 = 3.129 > 0 \Rightarrow [1.5, 1.75]$ (width 0.25). (c) $\gamma \approx 1.5 + \dfrac{0.25 \times 0.4375}{0.4375 + 3.129} = 1.5 + \dfrac{0.109}{3.567} = 1.531$ (3 s.f.; true root $1.5396\ldots$ – the interpolation sits just below it).

3.2 (a) $g(3) = 3\cos 1.5 + 1 = 1.2122 > 0$, $g(4) = 4\cos 2 + 1 = -0.6646 < 0$, and g is continuous on $[3, 4]$ (a product and sum of continuous functions), so the sign change gives a root $\beta \in [3, 4]$. (b) $\beta \approx 3 + \dfrac{(4-3) \times 1.2122}{1.2122 + 0.6646} = 3 + \frac{1.2122}{1.8768} = 3.6459 \approx 3.646$ (4 s.f.; true root $3.690\ldots$ – the curve bends away from the chord here, so one interpolation is only a first estimate).

Local review guidance: each stage of working shown with substitutions visible; conclusions stated as intervals with the continuity/sign-change justification; answers to the requested accuracy only. This is authored classroom guidance.