5.1 Complex numbers: arithmetic, Argand diagrams and polar form

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS complex number • conjugate • modulus • argument • Argand diagram

Complex numbers: arithmetic, Argand diagrams and polar form

Objective

Work confidently with complex numbers in the forms $a + bi$ and $r(\cos\theta + i\sin\theta)$: modulus, argument, conjugate, equality, sums, products and quotients, and their geometrical representation on an Argand diagram (FP1 1.1–1.3).

1 Key knowledge and worked examples

Modulus, argument, conjugate. For $z = a + bi$: $|z| = \sqrt{a^2 + b^2}$, $\bar{z} = a - bi$, and $\tan\theta = \frac{b}{a}$ with the quadrant chosen by the signs of a and b. Products multiply moduli: $|z_1 z_2| = |z_1|\,|z_2|$.

Worked example – a quotient with no fractions left. Simplify $\dfrac{4+7i}{2-3i}$: multiply numerator and denominator by the conjugate of the denominator, $\dfrac{(4+7i)(2+3i)}{2^2+3^2} = \dfrac{8+12i+14i+21i^2}{13} = \dfrac{-13+26i}{13} = -1+2i$. Check: $(2-3i)(-1+2i) = -2+4i+3i-6i^2 = 4+7i$ ✓.

Worked example – a real sum pins the imaginary part (WFM01 June 2025 Q5). $z_1 = r(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6})$ and z_2 has modulus 5. $|z_1 z_2| = 5r = 15$ gives $r = 3$, so $z_1 = -\frac{3\sqrt{3}}{2} - \frac{3}{2}i$. If $z_1 + z_2$ is real then $\mathrm{Im}(z_2) = \frac{3}{2}$, and $a^2 + \frac{9}{4} = 25$ gives $z_2 = \pm\frac{\sqrt{91}}{2} + \frac{3}{2}i \approx \pm 4.77 + 1.5i$ – one in the first quadrant, one in the second, both plotted 1.5 above the real axis.

2 Practice

2.1 Write each in the form $a + bi$: (a) $(3-2i)+(5+7i)$; (b) $(2+i)(4-3i)$; (c) $\dfrac{5+i}{3-i}$. [3]

2.2 Find the modulus and argument (exact, radians) of (a) $3+4i$; (b) $-2+2i$; (c) $-1-\sqrt{3}\,i$. [3]

2.3 Solve $x^2 + 4x + 29 = 0$, giving roots in the form $a \pm bi$. [2]

2.4 $z_1 = \sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$ and $z_2 = 2(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6})$. Find $|z_1 z_2|$ and hence write $z_1 z_2$ in polar form. [3]

2.5 On one Argand diagram plot $z = 2+3i$, $\bar{z}$, iz and $-z$, describing the geometry that links each pair. [3]

3 Exam-style questions

3.1 *(Original.)* The complex number w satisfies $|w| = 4$ and w has argument $\frac{2\pi}{3}$. (a) Write w exactly in the form $a + bi$. (b) Find $|w^2|$. (c) Mark w and w^2 on an Argand diagram, justifying the position of w^2 using moduli. [4]

3.2 *(Adapted from Edexcel IAL WFM01 June 2025 Q5.)* $z_1 = 3(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6})$ and $z_2 = a + bi$. Given that $|z_1 z_2| = 6$ and that $z_1 + z_2$ has zero imaginary part, determine the possible values of a and b. [5]