

5.1 Complex numbers: arithmetic, Argand diagrams and polar form

Name: _____ Class: _____ Date: _____

Total: 23 marks

KEY WORDS complex number • conjugate • modulus • argument
• Argand diagram

Complex numbers: arithmetic, Argand diagrams and polar form

Objective

Work confidently with complex numbers in the forms $a + bi$ and $r(\cos \theta + i \sin \theta)$: modulus, argument, conjugate, equality, sums, products and quotients, and their geometrical representation on an Argand diagram (FP1 1.1–1.3).

1 Key knowledge and worked examples

Modulus, argument, conjugate. For $z = a + bi$: $|z| = \sqrt{a^2 + b^2}$, $\bar{z} = a - bi$, and $\tan \theta = \frac{b}{a}$ with the quadrant chosen by the signs of a and b . Products multiply moduli: $|z_1 z_2| = |z_1| |z_2|$.

Worked example – a quotient with no fractions left. Simplify $\frac{4 + 7i}{2 - 3i}$: multiply numerator and denominator by the conjugate of the denominator, $\frac{(4 + 7i)(2 + 3i)}{2^2 + 3^2} = \frac{8 + 12i + 14i + 21i^2}{13} = \frac{-13 + 26i}{13} = -1 + 2i$. Check: $(2 - 3i)(-1 + 2i) = -2 + 4i + 3i - 6i^2 = 4 + 7i$ ✓.

Worked example – a real sum pins the imaginary part (WFM01 June 2025 Q5). $z_1 = r(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6})$ and z_2 has modulus 5. $|z_1 z_2| = 5r = 15$ gives $r = 3$, so $z_1 = -\frac{3\sqrt{3}}{2} - \frac{3}{2}i$. If $z_1 + z_2$ is real then $\text{Im}(z_2) = \frac{3}{2}$, and $a^2 + \frac{9}{4} = 25$ gives $z_2 = \pm \frac{\sqrt{91}}{2} + \frac{3}{2}i \approx \pm 4.77 + 1.5i$ – one in the first quadrant, one in the second, both plotted 1.5 above the real axis.

2 Practice

2.1 Write each in the form $a + bi$: (a) $(3 - 2i) + (5 + 7i)$; (b) $(2 + i)(4 - 3i)$; (c) $\frac{5 + i}{3 - i}$. [3]

2.2 Find the modulus and argument (exact, radians) of (a) $3 + 4i$; (b) $-2 + 2i$; (c) $-1 - \sqrt{3}i$. [3]

2.3 Solve $x^2 + 4x + 29 = 0$, giving roots in the form $a \pm bi$. [2]

2.4 $z_1 = \sqrt{2}(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$ and $z_2 = 2(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6})$. Find $|z_1 z_2|$ and hence write $z_1 z_2$ in polar form. [3]

2.5 On one Argand diagram plot $z = 2 + 3i$, $\bar{z}$, iz and $-z$, describing the geometry that links each pair. [3]

3 Exam-style questions

3.1 (*Original.*) The complex number w satisfies $|w| = 4$ and w has argument $\frac{2\pi}{3}$. (a) Write w exactly in the form $a + bi$. (b) Find $|w^2|$. (c) Mark w and w^2 on an Argand diagram, justifying the position of w^2 using moduli. [4]

3.2 (*Adapted from Edexcel IAL WFM01 June 2025 Q5.*) $z_1 = 3(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6})$ and $z_2 = a + bi$. Given that $|z_1 z_2| = 6$ and that $z_1 + z_2$ has zero imaginary part, determine the possible values of a and b . [5]
