

Solutions

2.1 (a) $8 + 5i$; (b) $8 - 6i + 4i - 3i^2 = 11 - 2i$; (c) $\frac{(5+i)(3+i)}{10} = \frac{15 + 5i + 3i + i^2}{10} = \frac{14 + 8i}{10} = \frac{7}{5} + \frac{4}{5}i$.

2.2 (a) $|z| = 5$, $\arg z = \arctan \frac{4}{3} \approx 0.927$; (b) $2\sqrt{2}$, $\frac{3\pi}{4}$; (c) 2 , $\tan \theta = \sqrt{3}$ with both parts negative so $\theta = -\frac{2\pi}{3}$ (or $\frac{4\pi}{3}$).

2.3 $(x+2)^2 + 25 = 0 \Rightarrow x = -2 \pm 5i$.

2.4 $|z_1 z_2| = \sqrt{2} \cdot 2 = 2\sqrt{2}$. In Cartesian form $z_1 = 1 + i$ and $z_2 = -\sqrt{3} + i$, so $z_1 z_2 = -(1 + \sqrt{3}) + (1 - \sqrt{3})i$ and $|z_1 z_2|^2 = (1 + \sqrt{3})^2 + (1 - \sqrt{3})^2 = 8$, confirming $2\sqrt{2}$.

2.5 z at $(2, 3)$; $\bar{z}$ at $(2, -3)$: reflection in the real axis; $iz = -3 + 2i$: quarter-turn anticlockwise about O ; $-z = -2 - 3i$: half-turn about O .

3.1 (a) $w = 4(-\frac{1}{2} + \frac{\sqrt{3}}{2}i) = -2 + 2\sqrt{3}i$. (b) $|w^2| = 16$. (c) w in the second quadrant on the circle of radius 4; w^2 on the circle of radius 16 – multiplying scales the modulus (and turns the point through $2 \times \frac{2\pi}{3} = \frac{4\pi}{3}$, equivalent to $-\frac{2\pi}{3}$, landing in the third quadrant).

3.2 $z_1 = 3(-\frac{\sqrt{3}}{2} + \frac{1}{2}i) = -\frac{3\sqrt{3}}{2} + \frac{3}{2}i$. Imaginary parts cancel: $b = -\frac{3}{2}$. Then $a^2 + \frac{9}{4} = |z_2|^2 = (6/3)^2 = 4$, so $a = \pm \frac{\sqrt{7}}{2}$: $z_2 = \pm \frac{\sqrt{7}}{2} - \frac{3}{2}i$.

Local review guidance: conjugate used to make a denominator real; exact modulus and argument values; Argand positions justified by geometry, not just plotted. This is authored classroom guidance.