## Solutions

**2.1**

$$1 + \tfrac{x}{3} - \tfrac{x^2}{9} + \cdots$$

Coefficient of $x^2$: $\frac{1}{3}(-\frac{2}{3}) \cdot \frac{1}{2} = -\frac{1}{9}$.

**2.2**

$$(2-x)^{-1} = \tfrac{1}{2}\left(1 - \tfrac{x}{2}\right)^{-1} = \tfrac{1}{2} + \tfrac{x}{4} + \tfrac{x^2}{8}, \qquad |x| < 2.$$

**2.3**

$$\sqrt{9+x} = 3\left(1 + \tfrac{x}{9}\right)^{1/2} = 3 + \tfrac{x}{6} - \tfrac{x^2}{216} + \cdots$$

**2.4** $x = 0.4$: $3 + 0.0667 - 0.00074 = 3.0659$; true $\sqrt{9.4} = 3.06594$ ✓.

**3.1** (a) From the worked example: $2 + \frac{x}{4} - \frac{x^2}{64}$. (b) Replace $x$ by $-x$: $2 - \frac{x}{4} - \frac{x^2}{64}$. (c) Divide (b)'s expansion into (a)'s:

$$\frac{2 + \frac{x}{4} - \frac{x^2}{64} + \cdots}{2 - \frac{x}{4} - \frac{x^2}{64} + \cdots} = \frac{1 + \frac{x}{8} - \frac{x^2}{128}}{1 - \frac{x}{8} - \frac{x^2}{128}} \approx \left(1 + \frac{x}{8}\right)\left(1 + \frac{x}{8}\right) \approx 1 + \frac{x}{4}$$

(to first order; the $x^2$ terms cancel to this accuracy: $\frac{x}{8} + \frac{x}{8} = \frac{x}{4}$ and $\frac{x^2}{64} - \frac{x^2}{64} - \frac{x^2}{64} = -\frac{x^2}{64}$, an order-2 remainder). With $x = 1$: $\sqrt{\frac{5}{3}} \approx 1.25$; true value $1.2909\ldots$ – a first-order approximation with visible error, improving with smaller $x$ ✓.

**3.2** $\dfrac{7-x}{(2-x)(1+x)} = \dfrac{A}{2-x} + \dfrac{B}{1+x}$: $7 - x = A(1+x) + B(2-x)$. At $x = 2$: $5 = 3A \Rightarrow A = \frac{5}{3}$. At $x = -1$: $8 = 3B \Rightarrow B = \frac{8}{3}$.

$$\frac{5}{3}(2-x)^{-1} + \frac{8}{3}(1+x)^{-1} = \frac{5}{6}\left(1 - \frac{x}{2}\right)^{-1} + \frac{8}{3}(1+x)^{-1}$$

$$= \frac{5}{6}\left(1 + \frac{x}{2} + \frac{x^2}{4} + \cdots\right) + \frac{8}{3}(1 - x + x^2 - \cdots) = \frac{7}{2} - \frac{9}{4}x + \frac{23}{8}x^2 + \cdots$$

valid for $|x| < 1$ (the tighter of $|x| < 2$ and $|x| < 1$). Check at $x = 0$: LHS $= \frac{7}{2}$; series $= \frac{5}{6} + \frac{8}{3} = \frac{7}{2}$ ✓.

Local review guidance: correct $n$ and $\frac{b}{a}$ scaling; simplified fractional coefficients; validity stated and approximations used within it. This is authored classroom guidance.