

Solutions

2.1 Suppose $\sqrt{3} = \frac{p}{q}$ in lowest terms. Then $3q^2 = p^2$, so p^2 is divisible by 3, so p is (if $p = 3m$, $p^2 = 9m^2$; conversely a prime dividing p^2 divides p). Write $p = 3m$: $3q^2 = 9m^2 \Rightarrow q^2 = 3m^2$, so q is divisible by 3 too – both divisible contradicts lowest terms. ■

2.2 $7x + 10 = A(x + 3) + B(x + 1)$: at $x = -1$, $A = \frac{3}{2}$; at $x = -3$, $B = \frac{11}{2}$. (Or compare coefficients: $A + B = 7$, $3A + B = 10$.)

2.3 With $\frac{5x + 7}{(x - 1)^2} = \frac{A}{x - 1} + \frac{B}{(x - 1)^2}$, multiply through: $5x + 7 = A(x - 1) + B$. Comparing x -coefficients, $A = 5$; comparing constants, $7 = -5 + B \Rightarrow B = 12$.

2.4 $\frac{x^2 + 2x + 5}{x^2 + x} = 1 + \frac{x + 5}{x^2 + x} = 1 + \frac{x + 5}{x(x + 1)}$; decompose $\frac{x + 5}{x(x + 1)} = \frac{5}{x} - \frac{4}{x + 1}$. So $A = 5$, $B = -4$. Check at $x = 1$: $1 + 5 - 2 = 4$; original $\frac{8}{2} = 4$ ✓.

3.1 Suppose some x in $90^\circ < x < 180^\circ$ has $|\sin x + \cos x| \geq 1$. Squaring: $1 + \sin 2x \geq 1$, so $\sin 2x \geq 0$. But $180^\circ < 2x < 360^\circ$ forces $\sin 2x < 0$ – a contradiction. So $|\sin x + \cos x| < 1$ throughout. ■ Check at $x = 135^\circ$: $|0| = 0 < 1$ ✓.

3.2 $\frac{2x^2 + 5x + 4}{(x + 2)(x + 1)^2} = \frac{A}{x + 2} + \frac{B}{x + 1} + \frac{C}{(x + 1)^2}$: $2x^2 + 5x + 4 = A(x + 1)^2 + B(x + 2)(x + 1) + C(x + 2)$.

- $x = -2$: $8 - 10 + 4 = 2 = A$. $x = -1$: $2 - 5 + 4 = 1 = C$. x^2 : $2 = A + B \Rightarrow B = 0$.

$$\frac{2}{x + 2} + \frac{1}{(x + 1)^2}$$

Check at $x = 1$: LHS = $\frac{2+5+4}{(1+2)(1+1)^2} = \frac{11}{12}$; RHS = $\frac{2}{3} + \frac{1}{4} = \frac{11}{12}$ ✓.

Local review guidance: assumption of the negation stated, or correct partial-fraction identity set up; exact values; verification or the named contradiction. This is authored classroom guidance.