

Solutions

2.1 $\frac{1}{5}e^{5x} + c.$

2.2 $3 \ln x + c.$

2.3 $\frac{1}{4} \sin 4x + c.$

2.4 Numerator $= (x^2 + 2x + 5)'$: $\ln(x^2 + 2x + 5) + c$ (the quadratic is always positive, so no modulus needed).

2.5 With $u = x^3 + 1$, $du = 3x^2 dx$: $\frac{1}{5}(x^3 + 1)^5 + c.$

2.6 $f(x) = x^3 - 3x + 1$: $f(0) = 1 > 0$, $f(1) = -1 < 0$; f is continuous (polynomial), and the sign change over $[0, 1]$ guarantees a root in $(0, 1)$.

3.1 (a) $\cos^2 2x = \frac{1}{2}(1 + \cos 4x)$:

$$\int \cos^2 2x \, dx = \frac{x}{2} + \frac{\sin 4x}{8} + c$$

Check: $\frac{1}{2} + \frac{4}{8} \cos 4x = \frac{1}{2}(1 + \cos 4x) = \cos^2 2x$ ✓. (b) Numerator $x = \frac{1}{2}(x^2 + 4)'$: $\frac{1}{2} \ln(x^2 + 4) + c.$

3.2 (a) $g(x) = x - e^{-x}$: $g(0.5) = 0.5 - 0.6065 = -0.1065 < 0$ and $g(0.6) = 0.6 - 0.5488 = 0.0512 > 0$; g is continuous, so a root lies in $(0.5, 0.6)$. (b) $x_2 = e^{-0.5} = 0.6065$; $x_3 = e^{-0.6065} = 0.545$ (3 d.p.). (c) $g(0.5665) = 0.5665 - e^{-0.5665} = 0.5665 - 0.5675 = -0.0010 < 0$ and $g(0.5675) = 0.5675 - 0.5669 = +0.0006 > 0$: the sign change between 0.5665 and 0.5675 fixes the root as 0.567 to 3 d.p. ✓ (True root: the omega constant 0.5671... ✓.)

Local review guidance: correct standard form or recognition of f' over f ; exact integrals with $+c$; sign-change statement naming continuity, and the iteration carried with enough steps. This is authored classroom guidance.