Solutions

2.1 $x = \frac{1}{2}\ln 50 = 1.96$ (3 s.f.).

2.2 $3x - 2 = \mathrm{e}^4$, so $x = \dfrac{\mathrm{e}^4 + 2}{3}$.

2.3 $P = 2000\mathrm{e}^{0.5} = 3297$; initial value 2000 (at $t = 0$).

2.4 $n = 3$ and $\log a = \log 6 \Rightarrow a = 6$: $y = 6x^3$.

2.5 $y = 10^{2x+1} = 10 \times 100^x$.

3.1 Gradient $= \dfrac{5-1}{0-(-2)} = 2$; intercept 5:

$$\log_{10} y = 2\log_{10} x + 5 \implies y = 10^5\, x^2 = 100\,000\, x^2$$

$p = 100\,000$, $q = 2$. Check at $(0, 5)$: $x = 1$, $y = 10^5$, $\log y = 5$ ✓.

3.2 (a) At $t = 0$: $95 = 25 + A \Rightarrow A = 70$. At $t = 6$: $40 = 25 + 70\mathrm{e}^{-6k} \Rightarrow \mathrm{e}^{-6k} = \frac{15}{70} = \frac{3}{14}$, so

$$k = \tfrac{1}{6}\ln\tfrac{14}{3} = 0.257 \text{ (3 s.f.)}$$

(b) $\dfrac{d\theta}{dt} = -70k\mathrm{e}^{-kt} = -4$:

$$\mathrm{e}^{-kt} = \frac{4}{70 \times 0.257} = 0.2226 \implies t = \frac{\ln(1/0.2226)}{0.257} = 5.85 \text{ (2 d.p.)}$$

Check: $\theta(5.85) = 25 + 70\mathrm{e}^{-0.257(5.85)} = 25 + 70(0.2226) = 40.6$ °C, falling at $70 \times 0.257 \times 0.2226 = 4.0$ °C/min ✓. (c) As $t \to \infty$, $\theta \to 25$ °C from above: the model's floor is room temperature, so it never predicts values below 25 °C – a colder-than-room prediction needs a new model.

Local review guidance: correct log recovery or correct differentiation of the model; exact/rounded values as asked; model interpretation tied to the limit. This is authored classroom guidance.

3.3

(a)

- $\ln N = \ln A + kt$ is a straight line with gradient $k = \frac{6.0-4.2}{6-0} = \mathbf{0.3}$ and intercept $\ln A = 4.2$, so $A = \mathrm{e}^{4.2} = \mathbf{66.7}$

(b)

- substitute $t = 10$: $N = A\mathrm{e}^{kt} = 66.69\,\mathrm{e}^{0.3(10)} = 66.69\,\mathrm{e}^3 = \mathbf{1340}$ (3 s.f.)

(c)

- set $N = 1000$ in $\ln N = 4.2 + 0.3t$: $0.3t = \ln 1000 - 4.2 = 2.7078$, so $t = \mathbf{9.03}$ hours

(d)

- growth cannot go on for ever: food and space run out