3.1 Functions: rational expressions, inverses

Name: _____ Class: _____ Date: _____ **Total: 26 marks**

KEY WORDS domain • range • composition • inverse function • rational expression

Objective

Simplify rational expressions, work with domain, range and composition of functions, and find and verify inverse functions. Original teaching questions for specification items P3.1.1 and P3.1.2; the exam-style part adapts reviewed WMA13 June 2025 demand.

1 Key knowledge and worked examples

- **Domain**: allowed inputs. **Range**: the outputs reached. **Composition** fg means do g first, then f.
- The **inverse** f^{-1} satisfies $f^{-1}f(x) = ff^{-1}(x) = x$; find it by writing $y = f(x)$ and solving for x.
- Simplify rational expressions by **factorising and cancelling factors** – never single terms.

Worked example – inverse with a rational function. $f(x) = \dfrac{3x}{2x-1}$, $x \neq \frac{1}{2}$. Find $f^{-1}(x)$ and state its domain.

- Write $y = \dfrac{3x}{2x-1}$ and solve for x:

$$y(2x-1) = 3x \implies 2xy - y = 3x \implies x(2y-3) = y \implies x = \frac{y}{2y-3}$$

$$f^{-1}(x) = \frac{x}{2x-3}, \qquad x \neq \tfrac{3}{2}$$

- Check: $f\left(\frac{x}{2x-3}\right) = \dfrac{3x/(2x-3)}{2x/(2x-3)-1} = \dfrac{3x}{2x-(2x-3)} = x$ ✓. The domain of f^{-1} is the range of f: as $x \to \infty$, $f(x) \to \frac{3}{2}$, so $f(x) \neq \frac{3}{2}$ ✓.

Worked example – simplify a rational expression. Simplify $\dfrac{x^2-9}{x^2+7x+12}$.

- Factorise top and bottom, then cancel the common factor:

$$\frac{(x-3)(x+3)}{(x+3)(x+4)} = \frac{x-3}{x+4} \qquad (x \neq -3, -4)$$

- Check at $x = 1$: original $\frac{-8}{20} = -\frac{2}{5}$; simplified $\frac{-2}{5}$ ✓.

2 Practice

2.1 Simplify $\dfrac{2x^2+5x-3}{x^2+2x-3}$. [3]

2.2 For $f(x) = \dfrac{x}{x+2}$ $(x \geq 0)$, state the range of f. [2]

2.3 With $f(x) = x^2$ and $g(x) = 2x+1$, find $fg(3)$ and state what you computed first. [2]

2.4 Find the inverse of $f(x) = 3x - 5$ and check it by composing. [3]

2.5 Find the inverse of $g(x) = \dfrac{x+1}{x-2}$, $x \neq 2$. [4]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA13 June 2025 Q1.)* $f(x) = \dfrac{2x}{3x+1}$, $x \geq 0$.

(a) Find the range of f.

(b) Find $f^{-1}(x)$.

(c) Solve $f^{-1}(x) = f(x)$. [6]

3.2 Express $\dfrac{5x+1}{x^2+x-2}$ as partial fractions, and hence solve $\dfrac{5x+1}{x^2+x-2} = 1$. [6]