

2.3 Coordinate geometry of the circle

Name: _____ Class: _____ Date: _____ Total: 25 marks

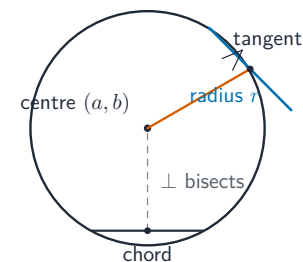
KEY WORDS chord • tangent • diameter • midpoint

Objective

Use the circle equation $(x - a)^2 + (y - b)^2 = r^2$ in both directions, complete the square on the expanded form, and apply the three examined circle properties. Original teaching questions for specification item P2.3.1; the exam-style part adapts reviewed WMA12 June 2025 demand.

1 Key knowledge and worked examples

- Centre (a, b) , radius r : $(x - a)^2 + (y - b)^2 = r^2$.
- Expanded form $x^2 + y^2 + fx + gy + c = 0$: complete both squares – centre $(-\frac{f}{2}, -\frac{g}{2})$, $r^2 = \frac{f^2 + g^2}{4} - c$.
- A **diameter** gives the centre as its **midpoint** and r^2 as a quarter of the diameter length squared.
- Properties: the angle in a semicircle is a right angle; the perpendicular from the centre to a **chord** bisects it; a **tangent** is perpendicular to the radius at the contact point.



Tangent $\perp$ radius at the contact point; the perpendicular from the centre to a chord **bisects** it; the angle in a semicircle is 90.

Tangent $\perp$ radius; centre-to-chord perpendicular bisects the chord.

Worked example – expanded form. Find the centre and radius of $x^2 + y^2 - 6x + 4y - 12 = 0$.

- Group and complete the square in x and in y :

$$(x^2 - 6x) + (y^2 + 4y) = 12 \implies (x - 3)^2 - 9 + (y + 2)^2 - 4 = 12$$

$$(x - 3)^2 + (y + 2)^2 = 25$$

- Centre $(3, -2)$, radius 5. Check with the point r right of the centre, $(8, -2)$: $64 + 4 - 48 - 8 - 12 = 0$ ✓, so it lies on the circle.

2 Practice

2.1 Find the centre and radius of $x^2 + y^2 + 8x - 2y + 8 = 0$. [3]

2.2 Write the equation of the circle with centre $(2, -5)$ and radius $\sqrt{13}$. [2]

2.3 Determine whether $(5, 2)$ lies on the circle $(x - 1)^2 + (y - 3)^2 = 17$. [2]

2.4 The points $(-3, 1)$ and $(5, 7)$ are the ends of a diameter. Find the circle's equation.^[4]

2.5 The circle $x^2 + y^2 = 10$ passes through $(3, 1)$. Find the equation of the tangent at that point. ^[4]

3 Exam-style questions

3.1 (*Adapted from Edexcel IAL WMA12 June 2025 Q2.*) The line joining $(-1, -3)$ and $(7, 1)$ is the diameter of circle C .

(a) Find an equation for C .

(b) Hence find the exact coordinates of the point on C that is nearest the x -axis. ^[7]

3.2 The circle $(x - 4)^2 + (y - 2)^2 = 25$ meets the line $x = 0$ at $A(0, 5)$ and $B(0, -1)$. Verify that the perpendicular from the centre to the chord AB bisects AB . ^[3]
