

Solutions

2.1 $(x + 4)^2 + (y - 1)^2 = 9$: centre $(-4, 1)$, radius 3.

2.2 $(x - 2)^2 + (y + 5)^2 = 13$.

2.3 $(5 - 1)^2 + (2 - 3)^2 = 16 + 1 = 17$ – yes, the point lies on the circle.

2.4 Centre: midpoint $= (1, 4)$. Diameter squared: $8^2 + 6^2 = 100$, so $r^2 = 25$.

$$(x - 1)^2 + (y - 4)^2 = 25$$

Check both endpoints: $16 + 9 = 25$ ✓ twice.

2.5 The radius to $(3, 1)$ has gradient $\frac{1}{3}$; the tangent's gradient is -3 :

$$y - 1 = -3(x - 3) \implies y = -3x + 10$$

Check: the tangent passes $(3, 1)$ ✓, and $-3 \times \frac{1}{3} = -1$ (perpendicular) ✓.

3.1 (a) Centre: $(\frac{-1+7}{2}, \frac{-3+1}{2}) = (3, -1)$. Diameter squared: $8^2 + 4^2 = 80$, so $r^2 = 20$:

$$(x - 3)^2 + (y + 1)^2 = 20$$

Check an endpoint: $(-4)^2 + 2^2 = 20$ ✓. (b) Nearest the x -axis means straight below the centre (the centre is 1 unit below the axis, less than $r = \sqrt{20} \approx 4.47$, so the circle crosses the axis; the lowest point is r below the centre):

$$(3, -1 - \sqrt{20}) = (3, -1 - 2\sqrt{5})$$

3.2 The centre is $(4, 2)$. The perpendicular from $(4, 2)$ to the vertical chord $x = 0$ is the horizontal line $y = 2$, meeting AB at $(0, 2)$. The midpoint of $A(0, 5)$ and $B(0, -1)$ is $(0, \frac{5+(-1)}{2}) = (0, 2)$ – the same point, so the perpendicular bisects the chord ✓.

Local review guidance: midpoint/completing-the-square method; exact centre, radius and equation; property applied with the right perpendicular. This is authored classroom guidance.