Solutions

2.1 $n = 3$: $9 + 1 = 10 = 2 \times 5$, which is not prime. One counterexample disproves the claim.

2.2 Let the integers be $(n-1)+n+(n+1) = 3n$, which is divisible by 3 for every integer n. ■

2.3 Non-multiples of 3 are $n = 3q + 1$ or $n = 3q + 2$ (complete case list).

- $n = 3q + 1$: $n^2 - 1 = 9q^2 + 6q = 3(3q^2 + 2q)$ ✓
- $n = 3q + 2$: $n^2 - 1 = 9q^2 + 12q + 3 = 3(3q^2 + 4q + 1)$ ✓ Both cases give a multiple of 3. ■

2.4 Let the integers be $2a+1$ and $2b+1$: their product is $4ab+2a+2b+1 = 2(2ab+a+b)+1$, which is one more than an even number – odd. ■

3.1 $m^2 + 3m + 2 = (m + 1)(m + 2)$. Cases: $m = 3q + 1$ or $m = 3q + 2$.

- $m = 3q + 1$: $(3q + 2)(3q + 3) = 3(3q + 2)(q + 1)$ – multiple of 3.
- $m = 3q + 2$: $(3q + 3)(3q + 4) = 3(q + 1)(3q + 4)$ – multiple of 3. Exhausting all non-multiples of 3, the expression is always a multiple of 3. ■ Check: $m = 1$: 6; $m = 2$: 12; $m = 4$: 30; $m = 5$: 42 – all divisible by 3 ✓.

3.2 A square's last digit depends only on its root's last digit. Exhaust the ten endings:

$$0^2 \to 0,\ 1^2 \to 1,\ 2^2 \to 4,\ 3^2 \to 9,\ 4^2 \to 6,\ 5^2 \to 5,\ 6^2 \to 6,\ 7^2 \to 9,\ 8^2 \to 4,\ 9^2 \to 1$$

The possible endings are $0, 1, 4, 5, 6, 9$ – never 8. ■

Local review guidance: complete case structure or a valid counterexample; algebra carried correctly; the conclusion states what was proved, with the case list justified. This is authored classroom guidance.