4.6 Vectors

Name: _____ Class: _____ Date: _____ **Total: 21 marks**

KEY WORDS magnitude • unit vector • position vector • scalar product

Objective

Work with vectors in two and three dimensions: magnitude and unit vectors, addition and scalar multiplication with their geometric meanings, position vectors, and angles from the scalar product. Original teaching questions for specification items P4.7.1 to P4.7.7; the exam-style part adapts reviewed WMA14 June 2025 demand.

1 Key knowledge and worked examples

- Column and **ijk** forms: $a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$.
- **Magnitude** : $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$; **unit vector** $\hat{\mathbf{a}} = \mathbf{a}/|\mathbf{a}|$.
- **Position vector** : $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.
- **Scalar product** : $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}||\mathbf{b}|\cos\theta$. Parallel vectors are scalar multiples; perpendicular ones have zero scalar product.

Worked example – magnitudes and a unit vector. $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

- $|\mathbf{a}| = \sqrt{4+1+4} = 3$, so the unit vector is $\dfrac{1}{3}(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$.
- Check: its magnitude is $\frac{1}{3}\sqrt{9} = 1$ ✓.

Worked example – a 45° angle (Edexcel IAL WMA14 June 2025 Q6). $A = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, $B = 5\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $C = 3\mathbf{i} + \alpha\mathbf{j} + 5\mathbf{k}$, angle $BAC = 45°$; find α.

- $\overrightarrow{AB} = 4\mathbf{i} + 4\mathbf{k}$ with $|\overrightarrow{AB}| = \sqrt{32}$; $\overrightarrow{AC} = 2\mathbf{i} + (\alpha - 2)\mathbf{j} + 8\mathbf{k}$; scalar product $= 8 + 0 + 32 = 40$.

$$\cos 45° = \frac{40}{\sqrt{32}\sqrt{68 + (\alpha-2)^2}} \implies \sqrt{68 + (\alpha-2)^2} = \frac{40\sqrt{2}}{4\sqrt{2}} = 10 \implies (\alpha-2)^2 = 32$$

$$\alpha = 2 \pm 4\sqrt{2}$$

- Check: $|AB||AC|\cos 45° = 4\sqrt{2} \times 10 \times \frac{1}{\sqrt{2}} = 40$ ✓.

2 Practice

2.1 Find $|3\mathbf{i} + 4\mathbf{j}|$ and the unit vector in that direction. [2]

2.2 For $P(1, 2, 3)$ and $Q(4, 6, 3)$, find $\overrightarrow{PQ}$ and its magnitude. [3]

2.3 Determine whether $\mathbf{a} = 2\mathbf{i} + (-1)\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = (-4)\mathbf{i} + 2\mathbf{j} + (-6)\mathbf{k}$ are parallel. [2]

2.4 Find the angle between $1\mathbf{i} + 1\mathbf{k}$ and $1\mathbf{j} + 1\mathbf{k}$, giving an exact value. [4]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA14 June 2025 Q6.)* Relative to O: A has position vector $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and B has position vector $3\mathbf{i} + \mathbf{j} + 6\mathbf{k}$. The point C has position vector $2\mathbf{i} + \beta\mathbf{j} + 4\mathbf{k}$, and angle BAC is $60°$. Find the exact possible values of β. [6]

3.2 The line segment RS has midpoint $M(1, 1, 0)$; R has position vector $3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$. Find $\overrightarrow{RS}$ and the distance RS. [4]

Solutions

2.1 Magnitude 5; unit vector $\frac{1}{5}(3\mathbf{i}+4\mathbf{j})$.

2.2 $\overrightarrow{PQ} = 3\mathbf{i}+4\mathbf{j}$, magnitude 5.

2.3 $\mathbf{b} = -2\mathbf{a}$: yes, parallel (one is a scalar multiple of the other).

2.4 Dot product $= 0+0+1 = 1$; magnitudes $\sqrt{2}$ each:

$$\cos\theta = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2} \implies \theta = 60^\circ$$

3.1 $\overrightarrow{AB} = 2\mathbf{i}+2\mathbf{j}+4\mathbf{k}$, $|AB| = \sqrt{24}$; $\overrightarrow{AC} = 1\mathbf{i}+(\beta+1)\mathbf{j}+2\mathbf{k}$; dot product $= 2+2(\beta+1)+8 = 2\beta+12$.

$$\cos 60^\circ = \frac{2\beta+12}{\sqrt{24}\sqrt{1+(\beta+1)^2+4}} = \frac{1}{2}$$

$$2(2\beta+12) = \sqrt{24}\sqrt{(\beta+1)^2+5} \implies 16(\beta+6)^2 = 24((\beta+1)^2+5)$$

$$2(\beta+6)^2 = 3((\beta+1)^2+5) \implies 2\beta^2+24\beta+72 = 3\beta^2+6\beta+18$$

$$\beta^2-18\beta-54 = 0 \implies \beta = \frac{18\pm\sqrt{540}}{2} = 9\pm 3\sqrt{15}$$

- Check $\beta = 9+3\sqrt{15} \approx 20.62$: dot $= 53.2$; $|AC| = \sqrt{1+467.4+4} = 21.7$; $\cos\theta = 53.2/(4.90\times 21.7) = 0.500 = \frac{1}{2}$ ✓ exactly.

3.2 $\overrightarrow{OR} = 3\mathbf{i}+(-1)\mathbf{j}+4\mathbf{k}$; $\overrightarrow{OM} = \frac{1}{2}(\overrightarrow{OR}+\overrightarrow{OS}) \Rightarrow \overrightarrow{OS} = 2\overrightarrow{OM}-\overrightarrow{OR} = (-1)\mathbf{i}+3\mathbf{j}+(-4)\mathbf{k}$.

$$\overrightarrow{RS} = \overrightarrow{OS}-\overrightarrow{OR} = (-4)\mathbf{i}+4\mathbf{j}+(-8)\mathbf{k}, \qquad RS = \sqrt{16+16+64} = \sqrt{96} = 4\sqrt{6}$$

- Check: M is the midpoint of R and S: $\frac{1}{2}(3+(-1)) = 1$, $\frac{1}{2}(-1+3) = 1$, $\frac{1}{2}(4+(-4)) = 0$ ✓.

Local review guidance: correct $\overrightarrow{AB}$, $\overrightarrow{AC}$ and scalar product; exact β or distance; angle equation set up with magnitudes. This is authored classroom guidance.