4.2 Parametric equations and differentiation

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

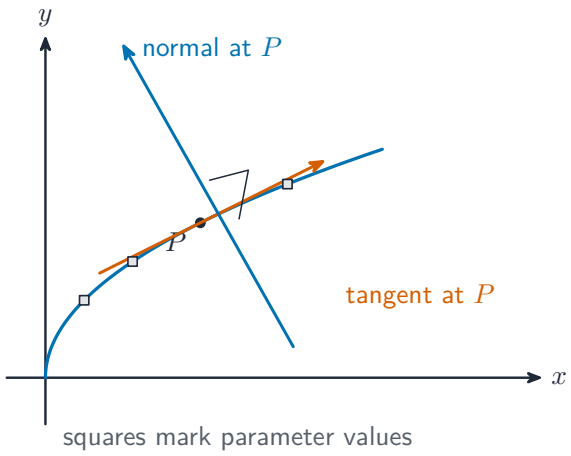
KEY WORDS parametric equations • Cartesian equation

Objective

Convert between parametric and Cartesian equations, differentiate parametric curves with $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$, and find tangents and normals. Original teaching questions for specification items P4.3.1 and P4.5.1; the exam-style part adapts reviewed WMA14 June 2025 demand.

1 Key knowledge and worked examples

- **Parametric** $x = f(t)$, $y = g(t)$: make t the subject of one and substitute to get the **Cartesian** equation (state excluded x).
- **Parametric differentiation**: $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$; tangent and normal gradients follow as usual.



Parametric gradient: $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$;
the normal takes its negative reciprocal.

Differentiate each of x and y with respect to t, then divide.

Worked example. $x = 2t$, $y = t^2 - 1$. Find $\dfrac{dy}{dx}$ and the tangent at $t = 2$.

- $\dfrac{dx}{dt} = 2$, $\dfrac{dy}{dt} = 2t$, so $\dfrac{dy}{dx} = \dfrac{2t}{2} = t$; at $t = 2$ the gradient is 2 and the point is $(4, 3)$.
- Tangent: $y - 3 = 2(x - 4) \implies y = 2x - 5$.
- Check: the Cartesian form is $y = \frac{x^2}{4} - 1$, whose gradient $\frac{x}{2}$ at $x = 4$ is 2 ✓.

Worked example – a normal from a paper point (verified facts from Edexcel IAL WMA14 June 2025 Q5). A parametric curve passes through $P(-7, -3)$ at $t = 2$ with $\dfrac{dy}{dx} = -\frac{4}{5}$ there.

- Normal gradient: $\dfrac{5}{4}$. Line: $y + 3 = \frac{5}{4}(x + 7)$, i.e.

$$5x - 4y + 23 = 0$$

- Check: $5(-7) - 4(-3) + 23 = 0$ ✓.

2 Practice

2.1 Convert $x = 3t$, $y = t^2$ to a Cartesian equation. [2]

2.2 For $x = t^2$, $y = 2t$ $(t > 0)$, find $\dfrac{dy}{dx}$ at $t = 3$. [3]

2.3 For $x = \cos t$, $y = \sin t$, find $\dfrac{dy}{dx}$ and state the curve. [3]

2.4 A curve has $x = t + 1$, $y = t^2 - 3t$. Find the equation of the tangent at $t = 2$. [4]

3 Exam-style questions

3.1 *(Demand adapted from Edexcel IAL WMA14 June 2025.)* A curve is given by $x = 4t$, $y = \dfrac{4}{t}$ $(t > 0)$.

(a) Find $\dfrac{dy}{dx}$ in terms of t.

(b) Find the equation of the normal at $t = 2$, in the form $ax + by + c = 0$ with integer coefficients. [6]

3.2 A curve has $x = t^2$, $y = t^3$. Find the Cartesian equation and the tangent at $t = -1$.[6]

Solutions

2.1 $t = \frac{x}{3} \Rightarrow y = \frac{x^2}{9}$.

2.2 $\dfrac{dx}{dt} = 2t = 6$, $\dfrac{dy}{dt} = 2$: $\dfrac{dy}{dx} = \frac{2}{6} = \frac{1}{3}$.

2.3 $\dfrac{dy}{dx} = \dfrac{\cos t}{-\sin t} = -\cot t$; eliminating t gives $x^2 + y^2 = 1$ (the unit circle).

2.4 $\dfrac{dy}{dx} = \dfrac{2t-3}{1}$; at $t = 2$: gradient 1, point $(3, -2)$. Tangent: $y + 2 = x - 3 \implies y = x - 5$.

3.1 (a) $\dfrac{dx}{dt} = 4$, $\dfrac{dy}{dt} = -\dfrac{4}{t^2}$:

$$\frac{dy}{dx} = -\frac{1}{t^2}$$

(b) At $t = 2$: gradient $-\frac{1}{4}$, point $(8, 2)$. Normal gradient 4: $y - 2 = 4(x - 8) \implies 4x - y - 30 = 0$. Check: $4(8) - 2 - 30 = 0$ ✓; the gradients multiply to $-\frac{1}{4} \times 4 = -1$ ✓.

3.2 Cartesian: $t^2 = x$, $t^3 = y \Rightarrow y^2 = t^6 = (t^2)^3 = x^3$, so $y^2 = x^3$. At $t = -1$: point $(1, -1)$, $\dfrac{dy}{dx} = \dfrac{3t^2}{2t} = \frac{3t}{2} = -\frac{3}{2}$. Tangent: $y + 1 = -\frac{3}{2}(x - 1) \implies 3x + 2y - 1 = 0$. Check: $3(1) + 2(-1) - 1 = 0$ ✓; and $(1, -1)$ satisfies $y^2 = x^3$ ✓.

Local review guidance: both derivatives with respect to t found; gradient value at the parameter; line through the correct point in the requested form. This is authored classroom guidance.