

2.7 Trigonometric identities and equations

Name: _____ Class: _____ Date: _____ **Total: 26 marks**

KEY WORDS identity • trigonometric equation

Objective

Use $\tan \theta \equiv \sin \theta / \cos \theta$ and $\sin^2 \theta + \cos^2 \theta \equiv 1$ to reduce trigonometric equations, and solve them in a given interval in degrees or radians. Original teaching questions for specification items P2.6.1 and P2.6.2; the exam-style part adapts reviewed WMA12 June 2025 demand.

1 Key knowledge and worked examples

Two identities, used to collapse an equation into one function:

$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta} \qquad \sin^2 \theta + \cos^2 \theta \equiv 1$$

Method: replace $\tan$ by $\sin / \cos$ (multiply through by $\cos$ if needed), replace $\sin^2$ by $1 - \cos^2$ (or the reverse), solve the (often quadratic) equation in one function, then read every solution in the given interval from the graph's symmetry. Keep the interval's units throughout.

Worked example – hidden quadratic. Solve $2 \sin^2 \theta = \cos \theta + 1$ for $0 \leq \theta < 360^\circ$.

- Replace $\sin^2 \theta$:
$$2(1 - \cos^2 \theta) = \cos \theta + 1 \implies 2 \cos^2 \theta + \cos \theta - 1 = 0 \implies (2 \cos \theta - 1)(\cos \theta + 1) = 0$$
- $\cos \theta = \frac{1}{2}$: $\theta = 60^\circ, 300^\circ$. $\cos \theta = -1$: $\theta = 180^\circ$.
- All solutions: $60^\circ, 180^\circ, 300^\circ$. Check one: $\theta = 60^\circ$: $2(0.75) = 1.5$ and $0.5 + 1 = 1.5$ ✓.

Worked example – factoring out. Solve $\tan x = \sin x$ for $0 \leq x \leq 360^\circ$.

- With $\tan x = \sin x / \cos x$:

$$\frac{\sin x}{\cos x} = \sin x \implies \sin x = \sin x \cos x \implies \sin x(1 - \cos x) = 0$$

- $\sin x = 0$: $x = 0^\circ, 180^\circ, 360^\circ$; $\cos x = 1$: $x = 0^\circ$ (already listed).
- Solutions: $0^\circ, 180^\circ, 360^\circ$. Check $x = 90^\circ$ is NOT a solution: $\tan 90^\circ$ undefined ✓ and dividing by $\cos x$ at the start would have lost $x = 90^\circ$ candidates – factor instead of dividing.

2 Practice

2.1 Simplify $\frac{\sin \theta \cos \theta}{\tan \theta}$. [2]

2.2 Solve $\sin \theta = -\frac{1}{2}$ for $0 \leq \theta < 360^\circ$. [3]

2.3 Solve $\tan 2\theta = 1$ for $0 \leq \theta < 180^\circ$. [4]

2.4 Given $\sin \theta = 0.6$ with θ acute, find $\cos \theta$ and $\tan \theta$. [3]

2.5 Solve $2 \cos^2 \theta + \cos \theta - 1 = 0$ for $0 \leq \theta < 360^\circ$. [4]

3 Exam-style questions

3.1 (Adapted from Edexcel IAL WMA12 June 2025 Q8(i).) Solve, for $0 \leq x < \pi$, the equation $\tan\left(2x - \frac{\pi}{4}\right) = \sqrt{3}$, giving the answers in radians in the form $k\pi$ where k is a rational constant to be found. [4]

3.2 (Adapted from the same question.) Solve, for $0 \leq \theta < 360^\circ$, the equation $4 \sin \theta \tan \theta = 3 \cos \theta$, giving your answers to one decimal place. [6]

Solutions

2.1 $\frac{\sin \theta \cos \theta}{\sin \theta / \cos \theta} = \cos^2 \theta.$

2.2 Third and fourth quadrants: $\theta = 180^\circ + 30^\circ = 210^\circ$ and $360^\circ - 30^\circ = 330^\circ.$

2.3 2θ runs over $[0^\circ, 360^\circ)$: $2\theta = 45^\circ$ or 225° , so $\theta = 22.5^\circ$ or $112.5^\circ.$

2.4 $\cos \theta = \sqrt{1 - 0.36} = 0.8$ (acute, so positive); $\tan \theta = 0.6/0.8 = 0.75.$

2.5 $(2 \cos \theta - 1)(\cos \theta + 1) = 0$: $\cos \theta = \frac{1}{2}$ gives $60^\circ, 300^\circ$; $\cos \theta = -1$ gives 180° . Solutions: $60^\circ, 180^\circ, 300^\circ.$

3.1 $\tan\left(2x - \frac{\pi}{4}\right) = \sqrt{3} = \tan \frac{\pi}{3}:$

$$2x - \frac{\pi}{4} = \frac{\pi}{3} + k\pi \implies 2x = \frac{7\pi}{12} + k\pi \implies x = \frac{7\pi}{24} + \frac{k\pi}{2}$$

In $0 \leq x < \pi$: $x = \frac{7\pi}{24}$ ($k = 0$) and $x = \frac{19\pi}{24}$ ($k = 1$). Check: at $x = \frac{7\pi}{24}$, $2x - \frac{\pi}{4} = \frac{\pi}{3}$ and $\tan \frac{\pi}{3} = \sqrt{3} \checkmark.$

3.2 Replace $\tan \theta$ and multiply through by $\cos \theta$:

$$\frac{4 \sin^2 \theta}{\cos \theta} = 3 \cos \theta \implies 4 \sin^2 \theta = 3 \cos^2 \theta \implies 4(1 - \cos^2 \theta) = 3 \cos^2 \theta \implies \cos^2 \theta = \frac{4}{7}$$

$$\cos \theta = \pm \frac{2}{\sqrt{7}} = \pm 0.7559$$

- $\cos \theta = 0.7559$: $\theta = 40.9^\circ$ and $360^\circ - 40.9^\circ = 319.1^\circ.$
- $\cos \theta = -0.7559$: $\theta = 180^\circ - 40.9^\circ = 139.1^\circ$ and $180^\circ + 40.9^\circ = 220.9^\circ.$ Solutions: $40.9^\circ, 139.1^\circ, 220.9^\circ, 319.1^\circ$ (1 d.p.). Check 40.9° : $\sin = 0.654$, $\tan = 0.865$: $4(0.654)(0.865) = 2.26$ and $3(0.756) = 2.27 \checkmark$ (rounding).

Local review guidance: identity applied to reach one function; correct root values; all interval solutions listed with quadrant reasoning. This is authored classroom guidance.