Exercise walk-through

7.3 Differentiating hyperbolic and inverse functions

Pearson Edexcel · International A-Level · Pure Mathematics

7. Integrating hyperbolic and inverse functions

Integrating

$\int \frac{dx}{\sqrt{x^2+a^2}} = \operatorname{arsinh} \frac{x}{a} + c$, $\int \frac{dx}{\sqrt{x^2-a^2}} = \operatorname{arcosh} \frac{x}{a} + c$, $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \arctan \frac{x}{a} + c$,
$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left|\frac{x-a}{x+a}\right| + c$. Complete the square first: $4x^2 + 8x + 9 = 4((x+1)^2 + \frac{5}{4})$.
Inverse functions: parts with u the inverse, e.g.
$\int \operatorname{arcosh} 3x \, dx = x \operatorname{arcosh} 3x - \frac{1}{3}\sqrt{9x^2 - 1} + c$.

10. Further matrix algebra

Matrices and eigenvalues

3×3 $\det$ by expansion; $\mathbf{A}^{-1} = \frac{1}{\det}(\text{adjugate})$; singular when $\det = 0$. Eigen check row by row: components of $\mathbf{Mv}$ equal λv_j. For $\begin{pmatrix} 1 & -1 & 4 \\ 3 & a & b \\ a & 1 & b \end{pmatrix}$ with eigenvector $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$: $\lambda = 3$, $a = 2$, $b = -1$, others 1 and -2. Symmetric matrices: orthogonal $\mathbf{P}$ of normalised eigenvectors, $\mathbf{P}^T\mathbf{AP}$ diagonal.

Check yourself

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Derive a cosh identity from exponentials; solve a mixed equation as a quadratic in e^x; state and prove an arsinh log form; place foci and directrices of both conics; tangent by parametric derivative and by discriminant; differentiate and integrate hyperbolic and inverse functions; build and iterate a reduction formula; arc length and surface exactly; cross and triple products; plane forms and skew distance; 3×3 inverse; eigenvalues, eigenvectors, diagonalisation.

Exercises 2.1·2.3

2.1 Differentiate $x \sinh 2x$. [4]

2.2 Differentiate $\cosh(3x^2 + 1)$. [3]

2.3 Differentiate $x^2 \operatorname{artanh} x$. [3]

Exercises 2.1·2.3

2.1 Differentiate $x \sinh 2x$. [4]

2.1 Product rule: $\sinh 2x + 2x \cosh 2x$.

2.2 Differentiate $\cosh(3x^2 + 1)$. [3]

2.2 Chain rule: $6x \sinh(3x^2 + 1)$.

2.3 Differentiate $x^2 \operatorname{artanh} x$. [3]

2.3 $2x \operatorname{artanh} x + \dfrac{x^2}{1 - x^2}$.

Exercises 2.4·3.1

2.4 Show that $\dfrac{d}{dx}\left(e^x \cosh x\right) = e^{2x}$. [3]

3.1 *(Edexcel IAL WFM03 June 2025 Q4.)* $y = \operatorname{arsinh} x + \operatorname{arsinh} \dfrac{1}{x}$, $x > 0$. (a) Show that $\dfrac{dy}{dx} = \dfrac{x-1}{x\sqrt{1+x^2}}$. (b) Hence determine the exact value of y for which $\dfrac{dy}{dx} = 0$, giving your answer as a simplified natural logarithm. [6]

Exercises 2.4·3.1

2.4 Show that $\dfrac{d}{dx}\left(e^x \cosh x\right) = e^{2x}$. [3]

2.4 $\dfrac{d}{dx}(e^x \cosh x) = e^x \cosh x + e^x \sinh x = e^x(\cosh x + \sinh x) = e^x \cdot e^x = e^{2x}$, since $\cosh x + \sinh x \equiv e^x$.

3.1 *(Edexcel IAL WFM03 June 2025 Q4.)* $y = \operatorname{arsinh} x + \operatorname{arsinh} \dfrac{1}{x}$, $x > 0$. (a) Show that $\dfrac{dy}{dx} = \dfrac{x-1}{x\sqrt{1+x^2}}$. (b) Hence determine the exact value of y for which $\dfrac{dy}{dx} = 0$, giving your answer as a simplified natural logarithm. [6]

3.1 (a) $\dfrac{d}{dx}\operatorname{arsinh} x = \dfrac{1}{\sqrt{1+x^2}}$ and, by the chain rule, $\dfrac{d}{dx}\operatorname{arsinh}\dfrac{1}{x} = \dfrac{-1/x^2}{\sqrt{1+1/x^2}}$. For $x > 0$, $\sqrt{1+1/x^2} = \dfrac{\sqrt{1+x^2}}{x}$, so the second term is $-\dfrac{1}{x\sqrt{1+x^2}}$ and $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1+x^2}}\left(1 - \frac{1}{x}\right) = \dfrac{x-1}{x\sqrt{1+x^2}}$. (b) $\dfrac{dy}{dx} = 0$ exactly at $x = 1$: $y = \operatorname{arsinh} 1 + \operatorname{arsinh} 1 = 2\ln(1+\sqrt{2})$.

Exercise 3.2

3.2 *(Original.)* (a) Differentiate $\operatorname{arcosh} 2x$. (b) Find the equation of the tangent to $y = \operatorname{arcosh} 2x$ at the point where $x = 1$, giving exact values. [6]

Exercise 3.2

3.2 *(Original.)* (a) Differentiate $\operatorname{arcosh} 2x$. (b) Find the equation of the tangent to $y = \operatorname{arcosh} 2x$ at the point where $x = 1$, giving exact values. [6]

3.2 (a) $\dfrac{d}{dx}\operatorname{arcosh} 2x = \dfrac{2}{\sqrt{4x^2-1}}$ (chain rule on $\sqrt{(2x)^2-1}$). (b) At $x = 1$: $y = \operatorname{arcosh} 2 = \ln(2+\sqrt{3})$ and the gradient is $\dfrac{2}{\sqrt{3}}$. Tangent: $y - \ln(2+\sqrt{3}) = \dfrac{2}{\sqrt{3}}(x-1)$.

Local review guidance: correct inverse derivative including the inner-function factor; the $x > 0$ simplification of $\sqrt{x^2}$ credited only when justified; exact values as logarithms or surds, never decimals. This is authored classroom guidance.