Exercise walk-through

6.4
Loci, regions and z→w transformations

Pearson Edexcel · International A-Level · Pure Mathematics

Exercises 2.1·2.3

2.1 Describe fully the locus given by $|z - 2i| = 3$. [2]

2.2 Describe fully the locus given by $|z - 3| = |z + 1|$. [2]

2.3 Show that the locus $|z - 2| = 2|z + 1|$ is a circle, and state its centre and radius. [4]

Exercises 2.1·2.3

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2.1 Circle, centre $2i$ (the point $(0, 2)$), radius 3.

2.2 Describe fully the locus given by $|z - 3| = |z + 1|$. [2]

2.2 Perpendicular bisector of the points 3 and -1: the vertical line $\operatorname{Re} z = 1$.

2.3 Show that the locus $|z - 2| = 2|z + 1|$ is a circle, and state its centre and radius. [4]

2.3 With $z = x+iy$: $(x-2)^2+y^2 = 4\left[(x+1)^2 + y^2\right]$, so $x^2-4x+4+y^2 = 4x^2+8x+4+4y^2$, i.e. $3x^2 + 12x + 3y^2 = 0$, so $x^2 + 4x + y^2 = 0$, i.e. $(x + 2)^2 + y^2 = 4$: circle, centre $(-2, 0)$, radius 2. (Check: $z = 0$ gives $|-2| = 2|1|$ ✓ and 0 lies on it.)

Exercises 2.4·3.1

2.4 Describe the locus given by $\arg\left(\dfrac{z-1}{z+1}\right) = \dfrac{\pi}{2}$. [3]

3.1 *(Original.)* The transformation T from the z-plane to the w-plane is given by $w = \dfrac{1}{z}$, $z \neq 0$. Show that T maps the circle $|z-2| = 2$ onto the straight line $\operatorname{Re} w = \frac{1}{4}$, and sketch this line. [5]

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2.4 $\arg\left(\frac{z-1}{z+1}\right) = \arg(z-1) - \arg(z+1)$ is the angle at z subtended by the segment from -1 to 1. Equal to $\frac{\pi}{2}$: the semicircle $x^2 + y^2 = 1$, $y > 0$, excluding the endpoints ± 1.

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3.1 $w = \frac{1}{z} \Rightarrow z = \frac{1}{w}$. Then $|\frac{1}{w} - 2| = 2$ gives $|1 - 2w| = 2|w|$. Squaring: $1 - 2w - 2\bar{w} + 4|w|^2 = 4|w|^2$, so $w + \bar{w} = \frac{1}{2}$, i.e. $\operatorname{Re} w = \frac{1}{4}$ – a vertical line in the w-plane, $\frac{1}{4}$ unit to the right of the imaginary axis. (Check: $z = 4$ is on the circle, and $z = 4 \mapsto w = \frac{1}{4}$ ✓.)

Exercise 3.2

3.2 *(Edexcel IAL WFM02 June 2025 Q3.)* The transformation T from the z-plane to the w-plane is given by $w = \dfrac{9i - iz}{1 + z}$, $z \neq -1$. The point P represents the complex number z, where $|z| = 1$. (a) Sketch the locus of P as z varies. (b) Given that the image under T of the locus of P, where $z \neq -1$, is the line l, determine, in simplest form, a Cartesian equation for l. (c) Write down the image under T of the point $z = i$ and verify that it lies on l. [9]

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3.2 (a) Unit circle, centre the origin. (b) $z = \dfrac{9i - w}{w + i}$, so $|9i - w| = |w + i| \cdot |z| = |w + i|$. With $w = u + iv$: $9i - w = -u + i(9 - v)$, so $u^2 + (9 - v)^2 = u^2 + (v + 1)^2$, giving $81 - 18v = 2v + 1$, i.e. $20v = 80$: l is $v = 4$. (c) $z = i$: $w = \dfrac{9i - i^2}{1 + i} = \dfrac{1 + 9i}{1 + i} = \dfrac{(1 + 9i)(1 - i)}{2} = \dfrac{10 + 8i}{2} = 5 + 4i$, whose imaginary part is 4, so it lies on l ✓.

Local review guidance: z made the subject before any modulus work; modulus equation expanded in u, v; line/circle description with geometry named (bisector, centre, radius). This is authored classroom guidance.