

Exercise walk-through

6.3

Euler's relation, de Moivre's theorem and complex roots

Pearson Edexcel · International A-Level · Pure Mathematics

Exercises 2.1·2.3

2.1 Use de Moivre's theorem to express $\cos 3\theta$ as a polynomial in $\cos \theta$. [3]

2.2 Express $\sin^3 \theta$ in the form $a \sin \theta + b \sin 3\theta$. [3]

2.3 Solve $z^3 = 8i$, giving the roots in the form $re^{i\theta}$ with exact θ . [4]

Exercises 2.1·2.3

2.1 Use de Moivre's theorem to express $\cos 3\theta$ as a polynomial in $\cos \theta$. [3]

2.1 Real part of $(c + is)^3$: $c^3 - 3cs^2 = c^3 - 3c(1 - c^2) = 4\cos^3 \theta - 3\cos \theta$.

2.2 Express $\sin^3 \theta$ in the form $a \sin \theta + b \sin 3\theta$. [3]

2.2 From $\sin 3\theta = 3s - 4s^3$ (imaginary part of $(c + is)^3$): $\sin^3 \theta = \frac{3}{4} \sin \theta - \frac{1}{4} \sin 3\theta$.

2.3 Solve $z^3 = 8i$, giving the roots in the form $re^{i\theta}$ with exact θ . [4]

2.3 $8i = 8e^{i\pi/2}$, so $z = 2e^{i(\pi/6+2\pi k/3)}$, $k = 0, 1, 2$: $2e^{i\pi/6}$, $2e^{i5\pi/6}$, $2e^{i3\pi/2}$. (Check: the product of the roots is $8e^{i(\pi/6+5\pi/6+3\pi/2)} = 8e^{i5\pi/2} = 8i$ ✓, matching $z_1 z_2 z_3 = -(-8i)$.)

Exercise 3.1

3.1 (*Original.*) (a) Show that $\cos^4 \theta = \frac{1}{8}(3 + 4 \cos 2\theta + \cos 4\theta)$. (b) Hence evaluate $\int_0^{\pi/2} \cos^4 \theta \, d\theta$. [7]

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3.1 (a) $\cos^4 \theta = \left(\frac{1+\cos 2\theta}{2}\right)^2 = \frac{1}{4} \left(1 + 2 \cos 2\theta + \frac{1+\cos 4\theta}{2}\right) = \frac{1}{8}(3 + 4 \cos 2\theta + \cos 4\theta)$. (Or: real part of $(\cos \theta + i \sin \theta)^4$ is $c^4 - 6c^2s^2 + s^4$; substitute $c^2 = \frac{1+\cos 2\theta}{2}$, $s^2 = \frac{1-\cos 2\theta}{2}$.) (b)

$$\int_0^{\pi/2} \frac{1}{8}(3 + 4 \cos 2\theta + \cos 4\theta) \, d\theta = \frac{1}{8} \left[3\theta + 2 \sin 2\theta + \frac{1}{4} \sin 4\theta\right]_0^{\pi/2} = \frac{1}{8} \cdot \frac{3\pi}{2} = \frac{3\pi}{16}.$$

Exercise 3.2

3.2 (*Edexcel IAL WFM02 June 2025 Q1.*) (a) Express $-2\sqrt{2}-2\sqrt{6}i$ in the form $r(\cos \theta + i \sin \theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$. (b) Hence solve $z^5 = -2\sqrt{2} - 2\sqrt{6}i$, giving your answers in the form $re^{i\theta}$, $r > 0$, $-\pi < \theta \leq \pi$. [8]

Exercise 3.2

3.2 (*Edexcel IAL WFM02 June 2025 Q1.*) (a) Express $-2\sqrt{2} - 2\sqrt{6}i$ in the form $r(\cos \theta + i \sin \theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$. (b) Hence solve $z^5 = -2\sqrt{2} - 2\sqrt{6}i$, giving your answers in the form $re^{i\theta}$, $r > 0$, $-\pi < \theta \leq \pi$. [8]

3.2 (a) Modulus $\sqrt{(-2\sqrt{2})^2 + (-2\sqrt{6})^2} = \sqrt{32} = 4\sqrt{2}$. Both components negative, and $\tan \theta = \frac{\sqrt{6}}{\sqrt{2}} = \sqrt{3}$ gives a reference angle $\frac{\pi}{3}$, so $\theta = -\pi + \frac{\pi}{3} = -\frac{2\pi}{3}$: $4\sqrt{2} \left(\cos \frac{2\pi}{3} \cdot (-1) \dots \right)$, i.e. $4\sqrt{2} \left(\cos(-\frac{2\pi}{3}) + i \sin(-\frac{2\pi}{3}) \right)$. (b) $z = (4\sqrt{2})^{1/5} e^{i \left(\frac{-2\pi/3 + 2\pi k}{5} \right)} = \sqrt{2} e^{i \left(-\frac{2\pi}{15} + \frac{2\pi k}{5} \right)}$, $k = 0, 1, 2, 3, 4$, i.e. arguments $-\frac{2\pi}{15}, \frac{4\pi}{15}, \frac{10\pi}{15} = \frac{2\pi}{3}, \frac{16\pi}{15} - 2\pi = -\frac{14\pi}{15}, \frac{22\pi}{15} - 2\pi = -\frac{8\pi}{15}$ (all in $-\pi < \theta \leq \pi$): $z \in \left\{ \sqrt{2}e^{-2\pi i/15}, \sqrt{2}e^{4\pi i/15}, \sqrt{2}e^{2\pi i/3}, \sqrt{2}e^{-14\pi i/15}, \sqrt{2}e^{-8\pi i/15} \right\}$.

Local review guidance: modulus and argument found exactly with the correct quadrant; roots carry the $\frac{2\pi k}{n}$ spread and land inside the stated argument range; power identities simplified with $c^2 + s^2 = 1$ substitutions. This is authored classroom guidance.