

Exercise walk-through

6.2

The method of differences

Pearson Edexcel · International A-Level · Pure Mathematics

Exercises 2.1·2.3

2.1 Show that $\sum_{r=1}^n \frac{1}{r(r+1)} = \frac{n}{n+1}$. [2]

2.2 Show that $\sum_{r=1}^n \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$. [4]

2.3 Show that $\sum_{r=1}^n r \cdot r! = (n+1)! - 1$. [3]

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2.1 $\frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1}$. The sum telescopes to $1 - \frac{1}{n+1} = \frac{n}{n+1}$.

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2.2 $\frac{1}{(2r-1)(2r+1)} = \frac{1}{2} \left[\frac{1}{2r-1} - \frac{1}{2r+1} \right]$ (check: $\frac{1}{2} \cdot \frac{(2r+1)-(2r-1)}{(2r-1)(2r+1)} = \frac{1}{(2r-1)(2r+1)}$ ✓).
 The sum is $\frac{1}{2} \left(1 - \frac{1}{2n+1} \right) = \frac{1}{2} \cdot \frac{2n}{2n+1} = \frac{n}{2n+1}$.

2.3 Show that $\sum_{r=1}^n r \cdot r! = (n+1)! - 1$. [3]

2.3 $r \cdot r! = (r+1-1)r! = (r+1)! - r!$. The sum telescopes to $(n+1)! - 1! = (n+1)! - 1$.
 (Check $n = 2$: $1 + 2 \cdot 2 = 5 = 6 - 1$ ✓.)

Exercise 3.1

3.1 (*Original.*) (a) Show that $\frac{1}{r(r+1)(r+2)} = \frac{1}{2} \left[\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)} \right]$. (b)

Hence find, in its simplest form, $\sum_{r=1}^n \frac{1}{r(r+1)(r+2)}$. [6]

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3.1 (a) $\frac{1}{2} \left[\frac{(r+2) - r}{r(r+1)(r+2)} \right] = \frac{1}{r(r+1)(r+2)} \checkmark$. (b) The sum is $\frac{1}{2} \left[\frac{1}{1 \cdot 2} - \frac{1}{(n+1)(n+2)} \right]$
 $\frac{1}{4} - \frac{1}{2(n+1)(n+2)}$. (Check $n = 1$: $\frac{1}{6}$ and $\frac{1}{4} - \frac{1}{12} = \frac{1}{6} \checkmark$.)

Exercise 3.2

3.2 (*Adapted from Edexcel IAL WFM02 June 2025 Q5.*) (a) Express $\frac{1}{(r+1)(r+2)}$ in partial fractions. (b) Hence show that $\sum_{r=1}^n \frac{1}{(r+1)(r+2)} = \frac{n}{2(n+2)}$. (c) Determine the smallest value of n for which the sum exceeds 0.49. [10]

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3.2 (a) $\frac{1}{(r+1)(r+2)} = \frac{1}{r+1} - \frac{1}{r+2}$. (b) Writing out $r = 1$ to n : $\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \dots + \left(\frac{1}{n+1} - \frac{1}{n+2}\right) = \frac{1}{2} - \frac{1}{n+2} = \frac{n}{2(n+2)}$. (c) $\frac{n}{2(n+2)} > 0.49 \Rightarrow n > 0.98(n+2) \Rightarrow 0.02n > 1.96 \Rightarrow n > 98$, so the smallest is $n = 99$ (check: $\frac{99}{202} \approx 0.4901 > 0.49$ ✓; $\frac{98}{200} = 0.49$ not strictly greater ✓).

Local review guidance: correct partial-fraction or factorial split verified; the cancellation displayed through first/last terms; closed form simplified and spot-checked. This is authored classroom guidance.